Unit 1
Constructions & Unknown Angles

Lesson 1: Construct an Equilateral Triangle

Opening Exercise

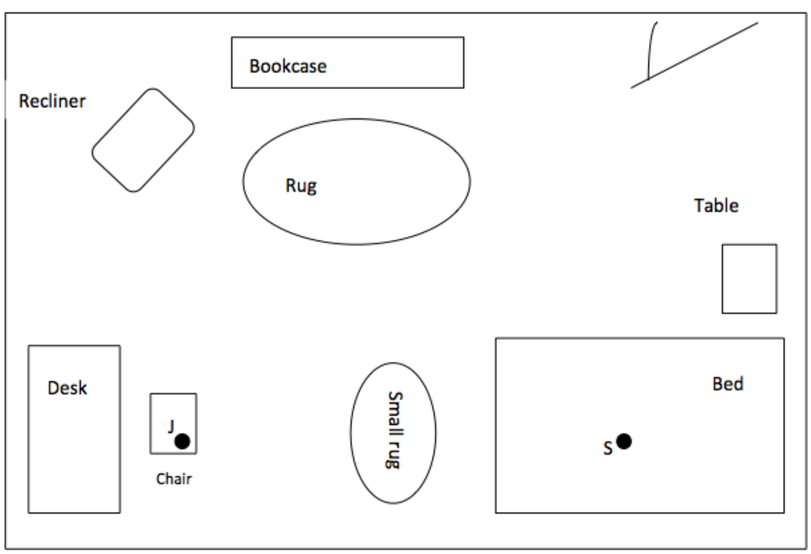
Joe and Marty are in the park playing catch. Tony joins them, and the boys want to stand so that the distance between any two of them is the same. Where do they stand?

How do they figure this out precisely? What tool or tools could they use?

Example 1

You will need a compass

Margie has three cats. She has heard that cats in a room position themselves at an equal distance from one another and wants to test that theory. Margie notices that Simon, her tabby cat, in the center of her bed (at S), while JoJo, her Siamese, is lying on her desk chair (at J). If the theory is true, where will she find Mack, her calico cat? Use the scale drawing of Margie's room shown below, along with the compass, and place an M where Mack will be if the theory is true.



What kind of shape have the cats formed? Be specific!!!!

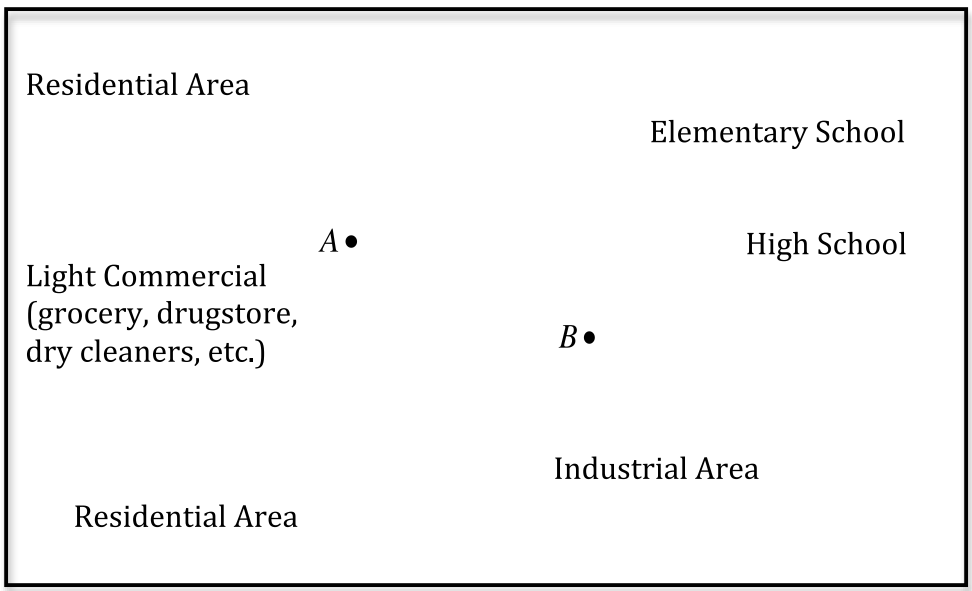
Vocabulary

Define	Diagram
Point • a location • named with a capital letter	
Line • one dimensional • goes on forever in both directions	
Segment • a measurable part of a line that consists of two endpoints and all the points between them.	
Ray • a line that has one endpoint and goes on forever in one direction	
Collinear • points that lie on the SAME LINE	
Plane • two dimensional • goes on forever in ALL direction	
Coplanar • points that lie in the SAME PLANE	
Circle • the locus of all points that are equidistant from a given point called the *center*	
Radius • a segment connecting the center of a circle to any point on the circle	

Example 2

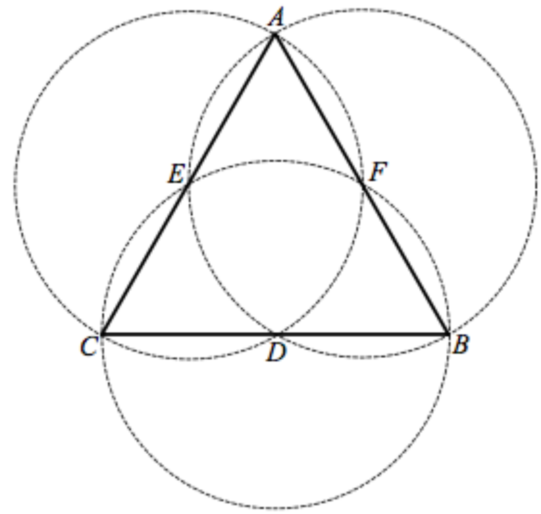
You will need a compass

Cedar City boasts two city parks and is in the process of designing a third. The planning committee would like all three parks to be equidistant from one another to better serve the community. A sketch of the city appears below, with the centers of the existing parks labeled as A and B. Identify two possible locations for the third park and label them as C and D on the map. Clearly and precisely list the mathematical steps used to determine each of the two potential locations.



Example 3

In the following figure, circles have been constructed so that the endpoints of the diameter of each circle coincide with the endpoints of each segment of the equilateral triangle.



a. What is special about points *D*, *E*, and *F*? Explain how this can be confirmed with the use of a compass.

b. Draw *DE*, *EF*, and *FD*. What kind of triangle must ΔDEF be?

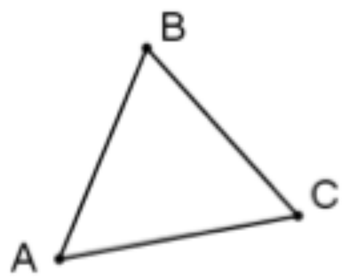
c. What is special about the four triangles within ΔABC?

d. How many times greater is the area of ΔABC than the area of ΔCDE?

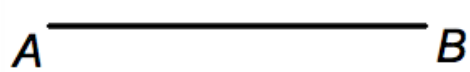
Homework

You will need a compass and a straightedge

1. ΔABC is shown below. Is it an equilateral triangle? Justify your response.



2. Construct equilateral triangle ABC using line segment $\overline{AB}$ as one side. Write a clear set of steps for this construction.

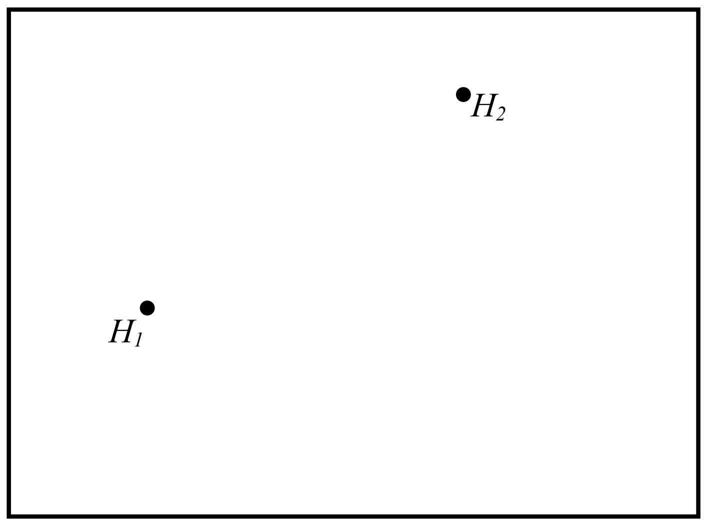


Lesson 2: Construct an Equilateral Triangle II

Opening Exercise

You will need a compass and a straightedge

Two homes are built on a plot of land. Both homeowners have dogs, and are interested in putting up as much fencing as possible between their homes on the land, but in a way that keeps the fence equidistant from each home. Use your construction tools to determine where the fence should go on the plot of land.



Example 1

You will need a compass and a straightedge

Using the skills you have practiced, construct **three** equilateral triangles, where the first and second triangles share a common side, and the second and third triangles share a common side.

If we were to continue using this procedure to construct 3 more equilateral triangles, what kind of figure would we have formed?

Example 2

You will need a compass and a straightedge

As a class, we are going to construct a hexagon inscribed in a circle, using the previous example as a guide.

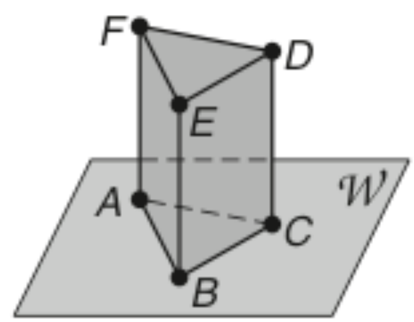
How could you connect the markings differently to make an inscribed equilateral triangle?

Postulates

In geometry, a postulate, or axiom, is a statement that describes a fundamental relationship between the basic terms of geometry. Postulates are accepted as true without proof.

Postulate	**Diagram**
Through any two points, there is exactly one line.	
Through any three non-collinear points there is exactly one plane.	
A line contains at least two points.	
A plane contains at least three non-collinear points.	
If two lines intersect, then their intersection is exactly one point.	
If two planes intersect, then their intersection is a line.	

In 1-4, use the diagram shown to the right:



1. How many planes are shown in the figure?

2. How many of the planes contain points *F* and *E*?

3. Name four points that are coplanar.

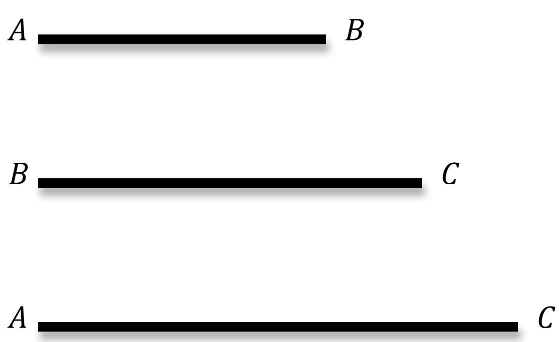
4. Are points *A*, *B* and *C* coplanar? Explain.

Homework

You will need a compass and a straightedge

1. Construct an equilateral triangle inscribed in a circle. (Use the hexagon construction in Example 2 as a guide!)

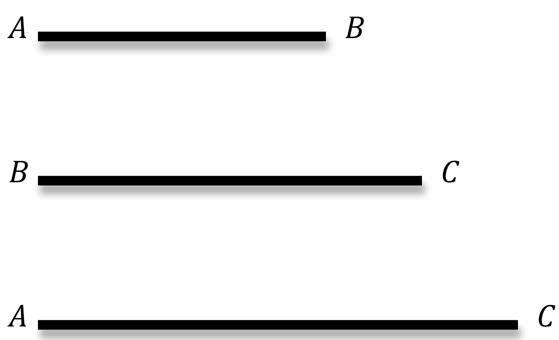
2. Construct scalene triangle ABC using the lengths of the 3 segments shown below:



Lesson 3: Copy and Bisect an Angle

Opening Exercise

Compare your homework answers with your partner. Shown below are the segments from Question #2. If needed, use the space provided to redraw scalene triangle *ABC*.



Vocabulary

Define	Diagram
Angle • The union of two non-collinear rays with the same endpoint. *In the diagram, identify the interior of the angle versus the exterior of the angle.*	
Convex • An angle that measures 180° or less	
Nonconvex • An angle that is greater than 180° but less than 360°	
Angle Bisector • A ray that divides an angle into two equal angles	
Segment Bisector • A segment, line or ray that divides a segment into two equal segments	
Linear Pair • A pair of adjacent angles whose non-common sides are opposite rays (supplemental angles)	
Degree • 1/360 of a circle	
Zero Angle • A ray and measures 0°	
Straight Angle • A line and measures 180°	
Midpoint • A point that is halfway between the endpoints of a segment	

Example 1

You will need a compass and a straightedge

Watch the video on how to bisect an angle:
http://www.mathopenref.com/constbisectangle.html

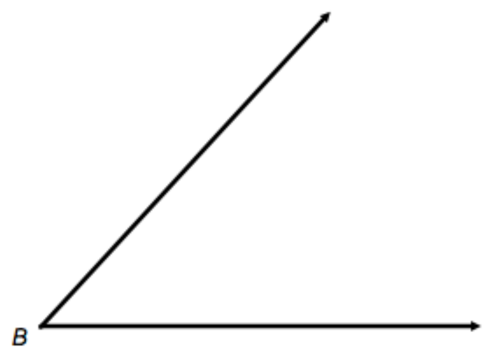
While watching the video, list the steps used to construct an angle bisector:

Using these steps, bisect the three angles shown below.   Revise your steps as needed.

Example 2

You will need a compass and a straightedge

Listed below are the steps needed to copy an angle. Carefully follow each step to complete your construction.

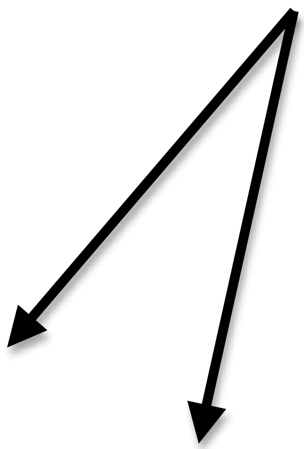


Draw ray EG (this will be the first side of the new angle)
Draw C_B: any radius (Label the intersection points A and C)
Draw C_E, same radius as C_B (Label the intersection point F)
Draw C_C : radius CA
Draw C_F : radius CA (Label the intersection D)
Draw ray ED

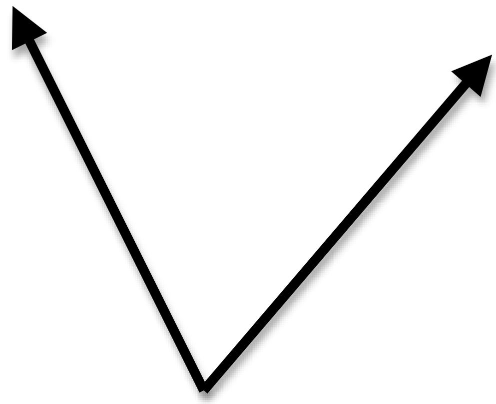
Homework

Using your compass and straightedge, bisect each angle below:

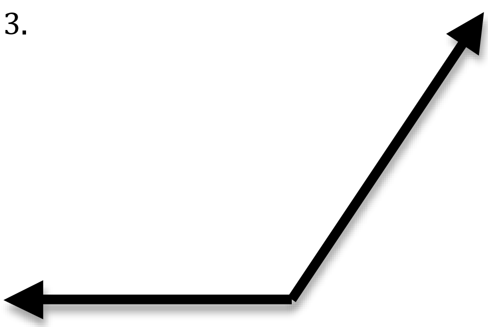
1.



2.



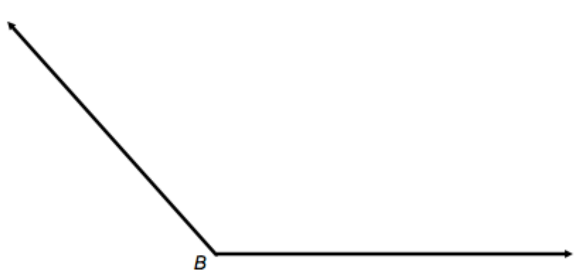
3.



4.

Using your compass and straightedge, copy the angle below:

5.



Lesson 4: Construct a Perpendicular Bisector

Opening Exercise

Hold the transparency over your homework. Did your angles and bisectors coincide perfectly?

Use the following rubric to evaluate your homework, marking which areas apply to you:

Needs Improvement	Satisfactory	Excellent
Few construction arcs visible	Some construction arcs visible	Construction arcs visible and appropriate
Few vertices or relevant intersections labeled	Most vertices and relevant intersections labeled	All vertices and relevant intersections labeled
Lines drawn without straightedge or not drawn correctly	Most lines neatly drawn with straightedge	Lines neatly drawn with straightedge
Fewer than 3 angle bisectors constructed correctly	3 of the 4 angle bisectors constructed correctly	Angle bisector constructed correctly

Vocabulary

Define	Diagram
Right Angle • an angle that measures $90°$	
Perpendicular • when two lines, segments or rays intersect forming a $90°$ angle	
Equidistant • a point is said to be equidistant when it is an equal distance from two or more things	

Define: Perpendicular Bisector

The *perpendicular bisector* of segment AB is the line ______________________________ to AB and passes through the ______________________________ of AB.

Example 1

You will need a compass and a straightedge

Thinking back to the fence problem in Lesson 2, experiment with your construction tools to determine the procedure to construct a perpendicular bisector.

Precisely describe the steps you took to bisect the segment.

Example 2

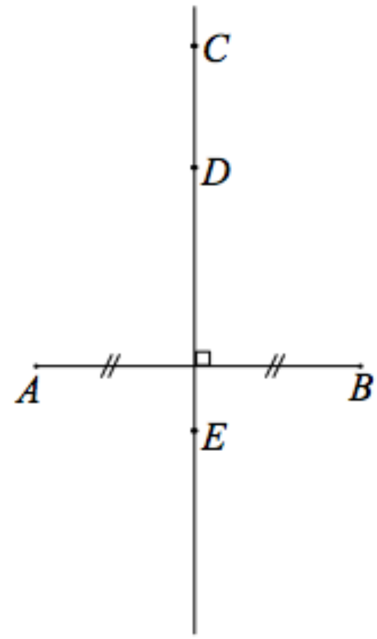
You will need a compass

Using your compass, examine the following pairs of segments:

a. *AC, BC*

b. *AD, BD*

c. *AE, BE*



Based on your findings, fill in the observation below:

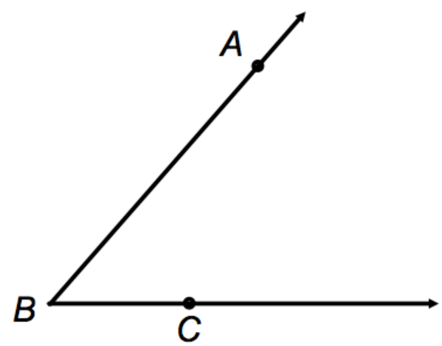
Any point on the perpendicular bisector of a line segment is ______________________________ from the endpoints of the line segment.

Example 3

You will need a compass and a straightedge

Using the diagram pictured below, we will perform the following steps:

1. Trace $\angle ABC$ on a separate sheet of paper, including points *A*, *B* and *C*.
2. Using your compass and straightedge, construct the angle bisector.
3. Fold your paper along the angle bisector and plot the two reflection points. Name point *A*'s reflection *E*, and name point *C*'s reflection *D*.
4. Unfold your paper and using a ruler draw segments *AE* and *CD*.
5. Trace these new points, segments, and angle bisector on to the original diagram pictured below.



What is the relationship between these newly formed segments and the angle bisector?

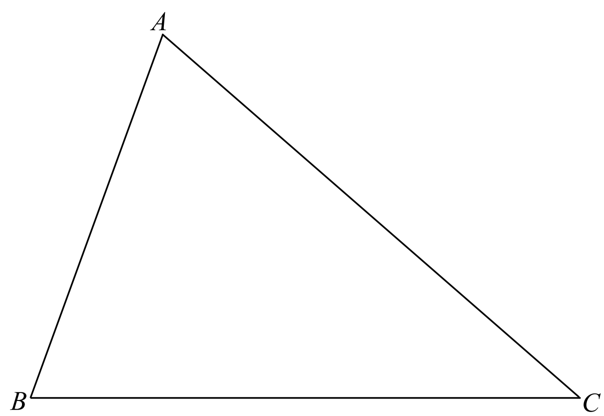
Now using your compass measure the length of segment *DC*. Using this length, construct two circles, one with a center at *D* and one with a center at *C*.

What do you notice about the intersection points of the constructed circles?

Homework

You will need a compass and a straightedge

Construct the perpendicular bisectors of *AB*, *BC* and *CA* on the triangle below. What do you notice about the segments you have constructed?

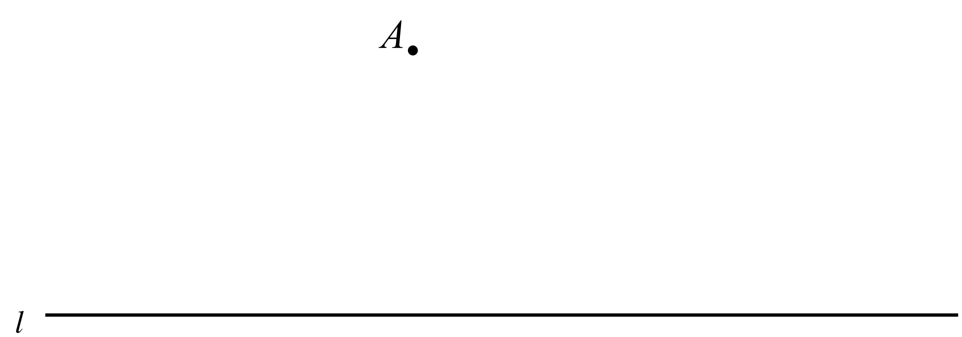


Lesson 5: Construct a Perpendicular Bisector II

Opening Exercise

You will need a compass and a straightedge

You know how to construct the perpendicular bisector of a segment. Now you will investigate how to construct a perpendicular to a line l from a point A not on l. Think about how you have used circles in constructions so far and *why* the perpendicular bisector construction works the way it does. The first step of the instructions has been provided for you. Discover the construction and write the remaining steps.



Steps:

1. Draw a circle A so that the circle intersects line l in two points. Label these points B and C.

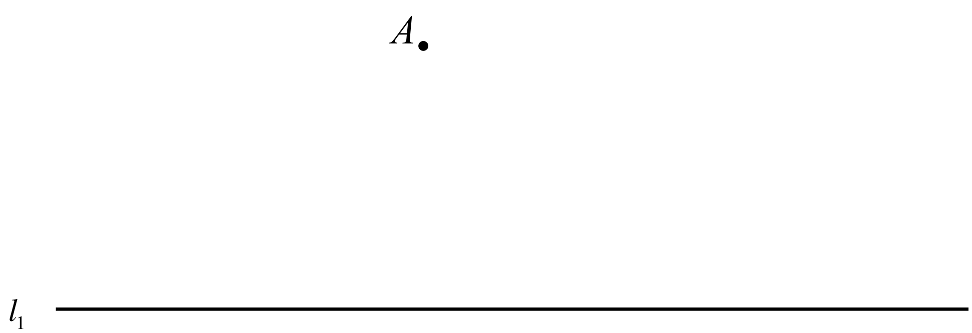
Example 1

You will need a compass and a straightedge

You are going to use the concept of constructing perpendicular lines to construct parallel lines!

Step 1: Construct a line perpendicular to l_1 through point *A.* Call this new line l_3.

Step 2: Construct a line perpendicular to l_3 through point *A*. Call this new line l_2.



Vocabulary

Define	Diagram
Parallel Lines	

In the construction above, you have also constructed a square inscribed in a circle. Can you find it?

Example 2

You will need a compass and a straightedge

What are the characteristics of a square?

Using these characteristics and your knowledge of constructions, we are going to construct a square.

Homework

You will need a compass and a straightedge

An **isosceles triangle** is a triangle that has two congruent sides.

An **isosceles right triangle** has a right angle between the two congruent sides (these sides are perpendicular!).

Using the segment below as one of your congruent sides, construct an isosceles right triangle. List your steps below:

Steps:

Lesson 6: Points of Concurrencies

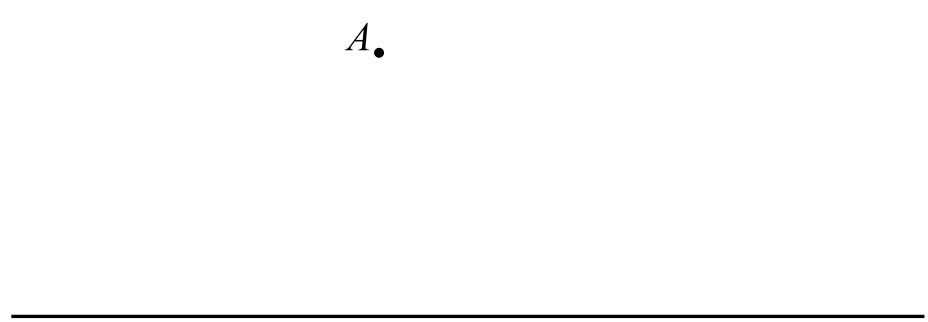
Opening Exercise

You will need a compass and a straightedge

We are now going to look at a second way of constructing parallel lines using another construction we are familiar with … copying an angle!

We will walk through the procedure as a class, writing down our steps as we go.

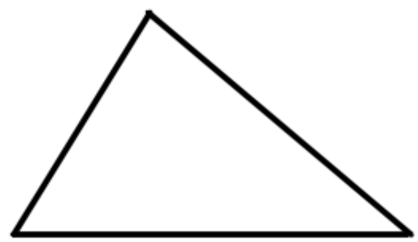
Construct a line parallel to the given line going through point *A:*



Example 1

You will need a compass and a straightedge

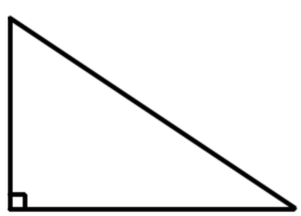
Sketch the perpendicular bisector for each side of the triangle pictured below.

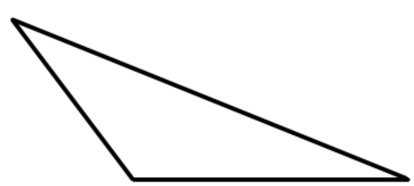


Vocabulary

- When 3 or more lines intersect in a single point they are _____________________________.
- This point of intersection is called the ____.
- The point of intersection for 3 perpendicular bisectors is called the ____________________.

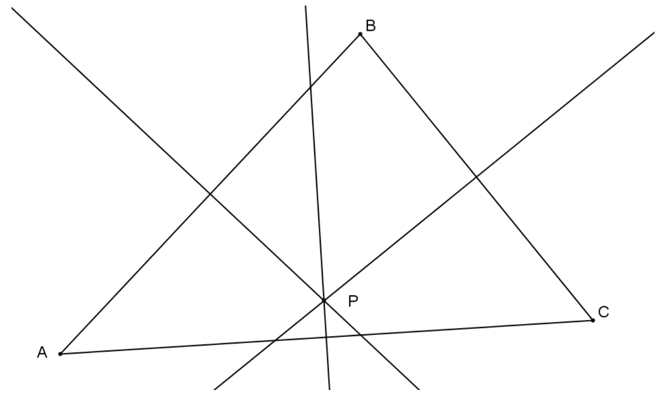
We will use http://www.mathopenref.com/trianglecircumcenter.html to explore what happens when the triangle is right or obtuse. Sketch the location of the circumcenter on the triangles below:





Example 2

Using the picture to the right, mark the right angles and congruent segments if point P is the circumcenter of ΔABC.



Look back at Example 3 from Lesson 4.

We discovered that: Any point on the perpendicular bisector of a line segment is ______________________________ from the endpoints of the line segment.

Based on this discovery what could you conclude about:

AP and BP?

BP and CP?

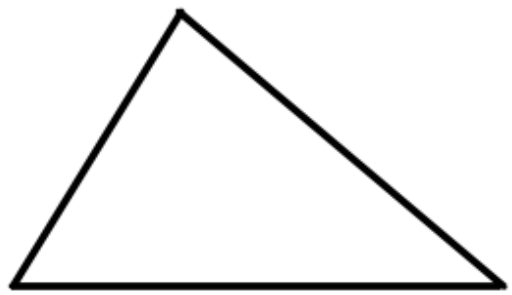
CP and AP?

This tells us that the perpendicular bisectors will always be concurrent!

Example 3

You will need a compass and a straightedge

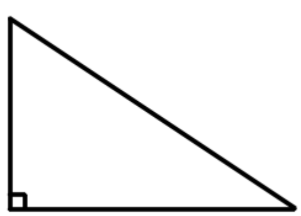
Sketch the angle bisectors for each angle of the triangle pictured below.

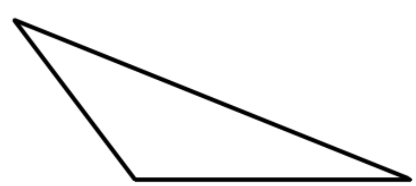


Vocabulary

- The point of intersection for 3 angle bisectors is called the ________________.

We will use http://www.mathopenref.com/triangleincenter.html to explore what happens when the triangle is right or obtuse. Sketch the location of the incenter on the triangles below:





Homework

1. How many points determine a line?

2. How many points determine a plane? What must be true about the points?

3. Two non-parallel lines intersect how many times?

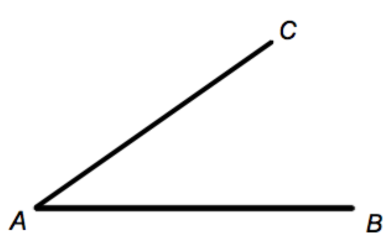
4. The intersection of two planes is what kind of figure?

5. Using a compass and straightedge, construct the following:

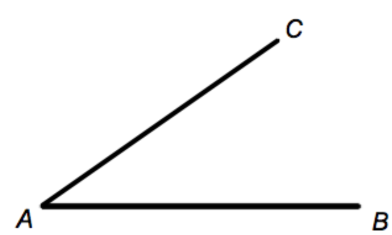
   a. Equilateral Triangle

   b. Perpendicular Bisector

   c. Angle Bisector



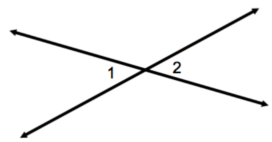
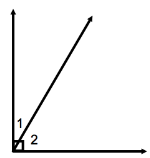
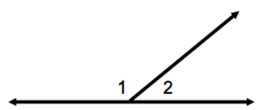
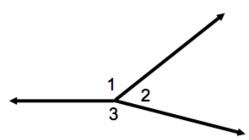
   d. Copy the given angle



Lesson 7: Solve for Unknown Angles – Angles and Lines at a Point

Opening Exercise

Fill in the "Fact/Discovery" column based on geometry facts you have learned in the past!

Name	Diagram	Fact/Discovery
Vertical Angles		
Angles forming a right angle		
Angles on a Line		
Angles at a Point		

Define:

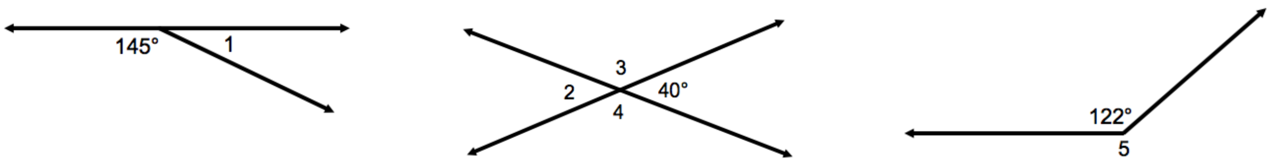
<u>Complementary</u>:

<u>Supplementary</u>:

<u>Adjacent</u>:

Example 1

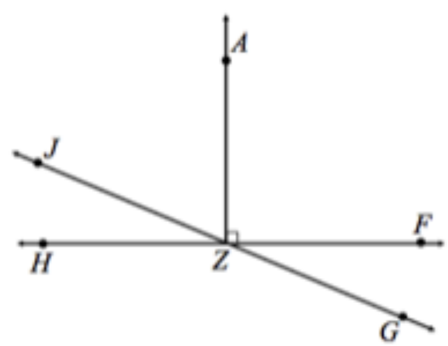
Find the measure of each labeled angle. Give a reason for your solution.



Angle	Angle Measure	Reason
$\angle 1$		
$\angle 2$		
$\angle 3$		
$\angle 4$		
$\angle 5$		

Example 2

Use the following diagram pictured to the right to answer the following:



a. Name an angle supplementary to $\angle HZJ$ and provide the reason.

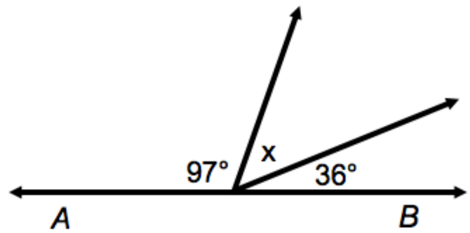
b. Name an angle complementary to $\angle HZJ$ and provide the reason.

c. Name an angle congruent to $\angle HZJ$ and provide the reason.

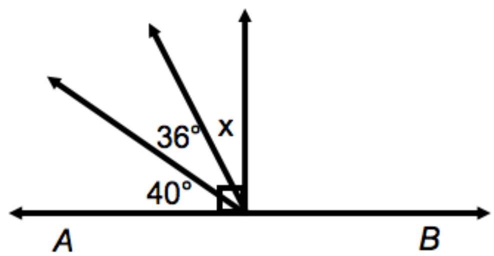
Exercises

Find the value of *x* and/or *y* in each diagram below. Show all steps to your solution.

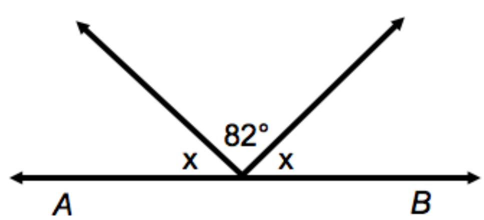
1.



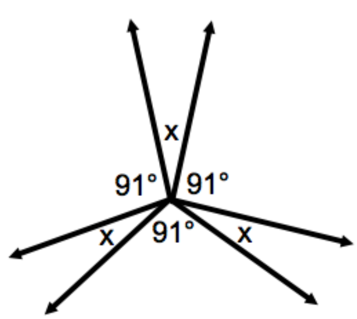
2.



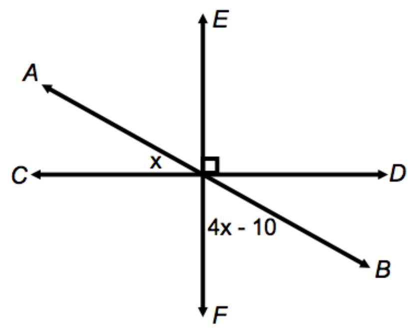
3.



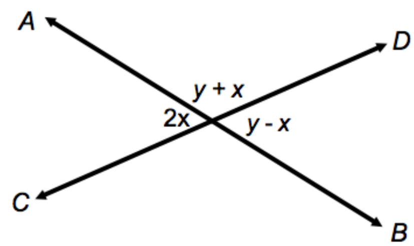
4.



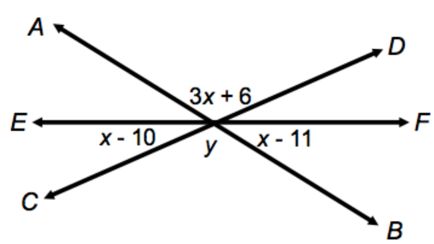
5.


E
A
x
C
D
4x - 10
B
F

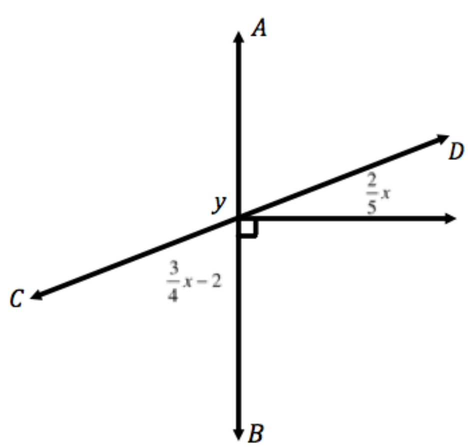
6.


A
D
y + x
2x
y - x
C
B

7.


A
D
3x + 6
E
F
x - 10
y
x - 11
C
B

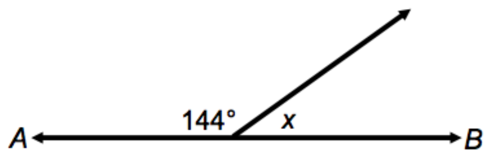
8.


A
D
y
$\frac{2}{5}x$
$\frac{3}{4}x-2$
C
B

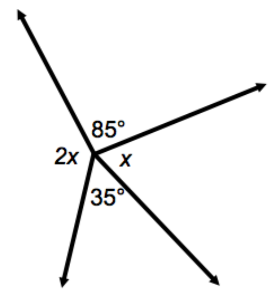
Homework

In the figures below, *AB* and *CD* are straight lines. Find the value of *x* and/or *y* in each diagram below. Show all steps to your solution.

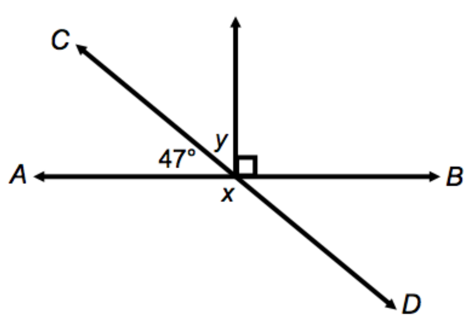
1.



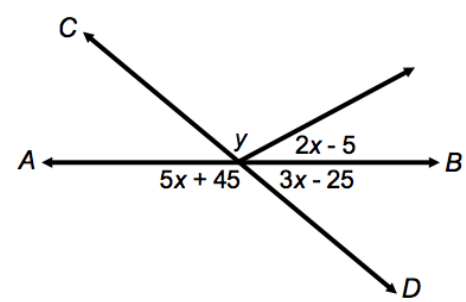
2.



3



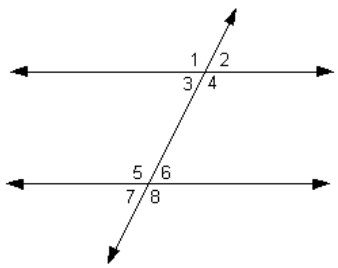
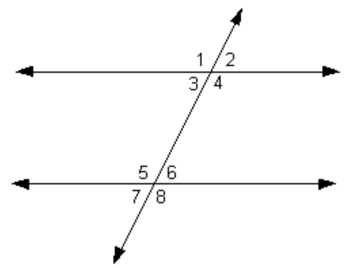
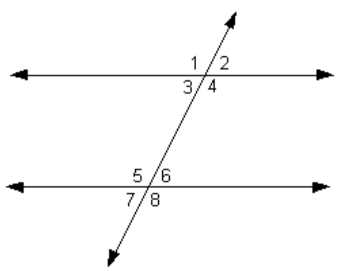
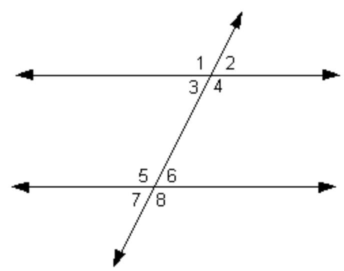
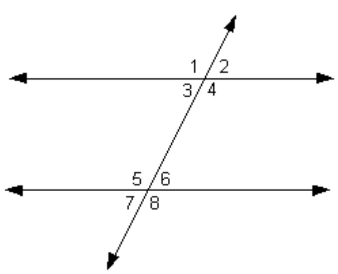
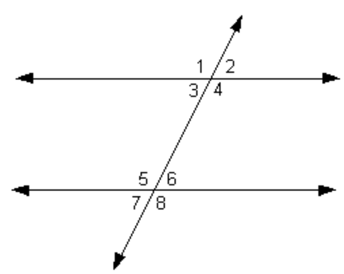
4.



Lesson 8: Solve for Unknown Angles – Transversals

Opening Exercise

With your partner, identify the following:

Interior Angles 	**Exterior Angles** 
Alternate Interior Angles 	**Alternate Exterior Angles** 
Corresponding Angles 	**Same Side Interior Angles** 

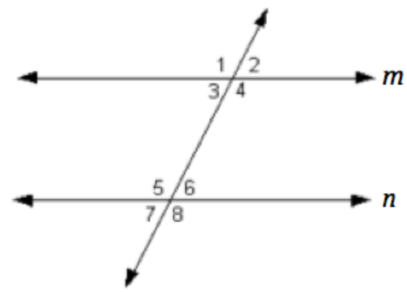
<u>Properties of parallel lines cut by transversal:</u>

- The alternate interior angles are **congruent**.
- The corresponding angles are **congruent**.
- The same side interior angles are **supplementary**.

Example 1

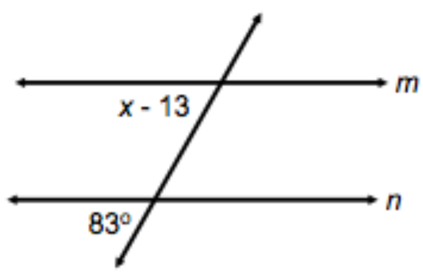
If $m \parallel n$ and $m\angle 1 = 150°$, find the measure of the remaining angles and provide your reasoning.

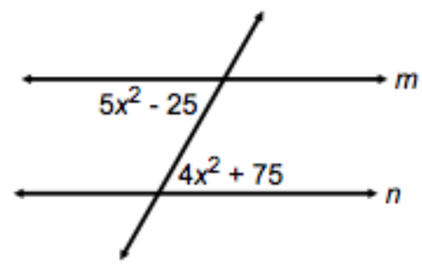
	Angle Measure	Reasoning
$m\angle 2$		
$m\angle 3$		
$m\angle 4$		
$m\angle 5$		
$m\angle 6$		
$m\angle 7$		
$m\angle 8$		

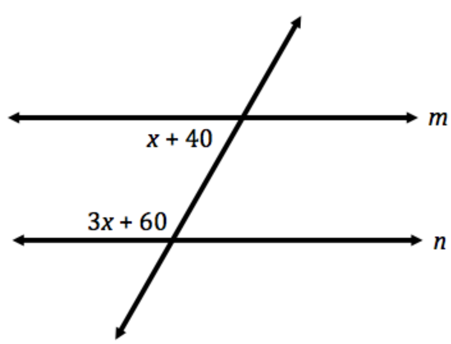


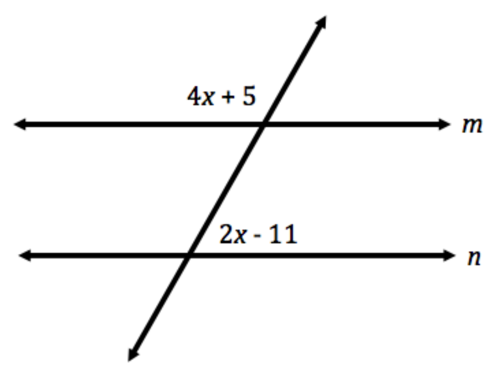
Example 2

If $m \parallel n$, find the value of x for the problems pictured below.





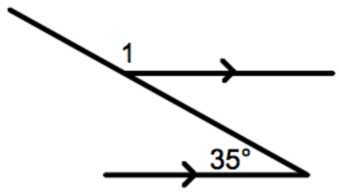




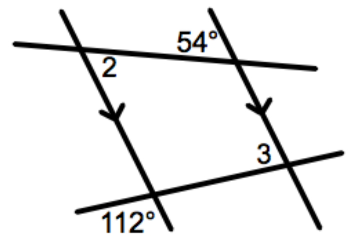
Exercises

Find the values of the missing (labeled) angles in each diagram below. Show all steps to your solutions.

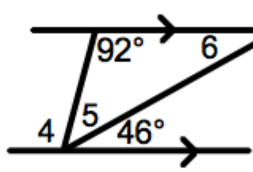
1.



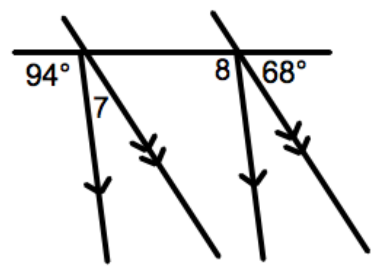
2.



3.



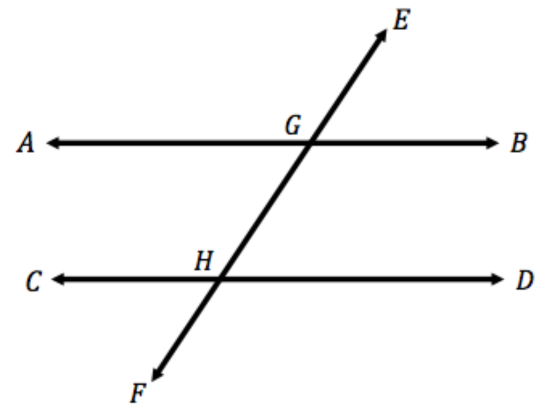
4.



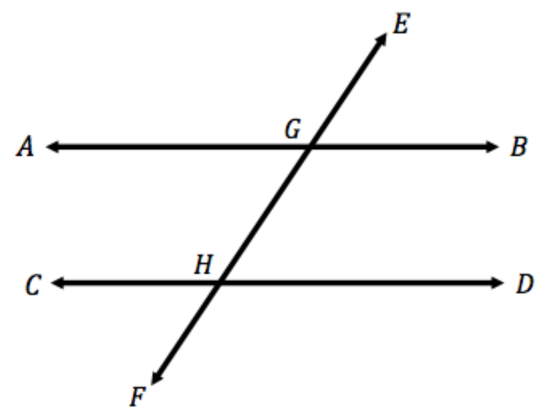
Homework

In 1-3, $AB \parallel CD$ and are intersected by transversal EF at G and H.

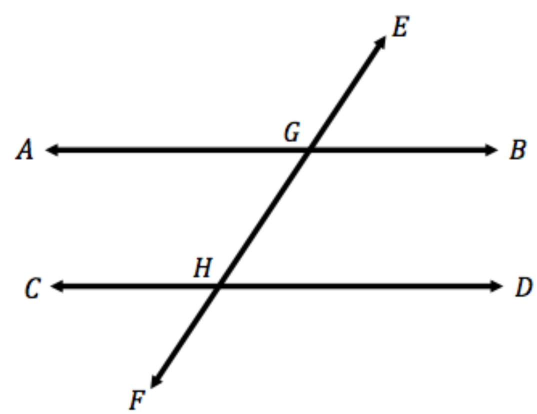
1. If $m\angle EGB = 40°$, find the remaining angles.



2. If $\angle AGH = x + 40$ and $\angle CHG = 3x + 60$, find x.

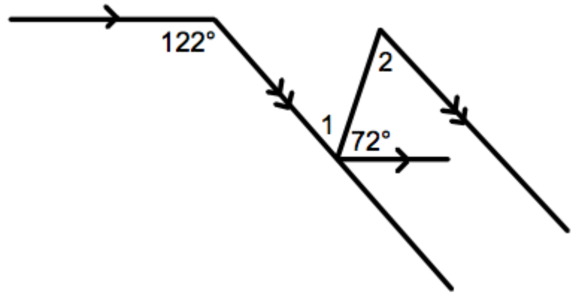


3. If $\angle AGH = 3x - 10$ and $\angle DHG = 7x - 42$, find $\angle DHG$.

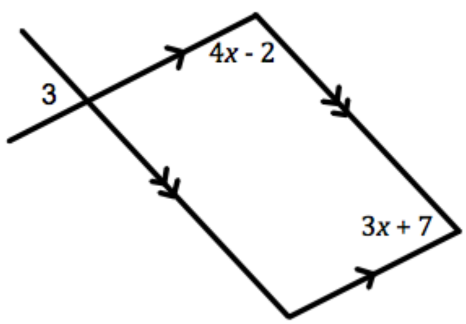


In 4-5, find the measure of the missing (labeled) angles. Show all work!

4.



5.

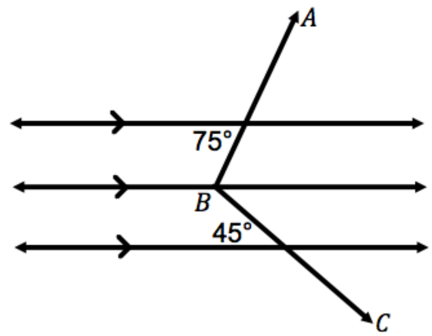


Lesson 9: Solve for Unknown Angles – Auxiliary Lines

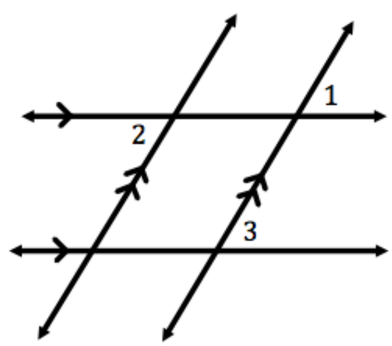
Opening Exercise

Using your knowledge of parallel lines, answer the following:

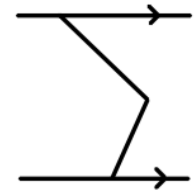
1. What is the measure of $\angle ABC$?



2. If $m\angle 1 = 16x - 8$, $m\angle 2 = 4(y + 8)$, and $m\angle 3 = 14x + 2$, find x and y.



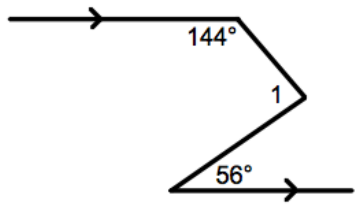
Vocabulary

Define	Diagram
Auxiliary Line	

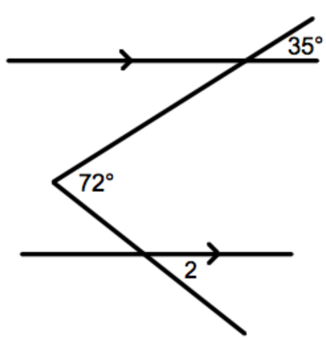
Exercises

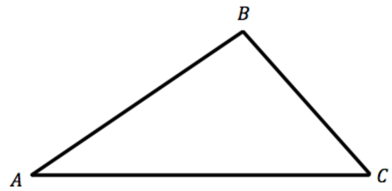
Use auxiliary lines to find the unknown (labeled) angles. Show all work!

1.



2.

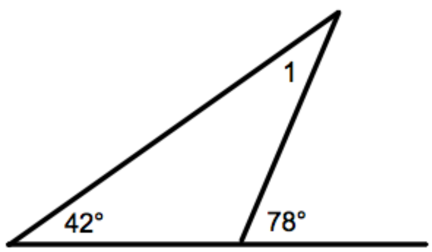


Name	Diagram	Fact/Discovery
∠ Sum of Δ		

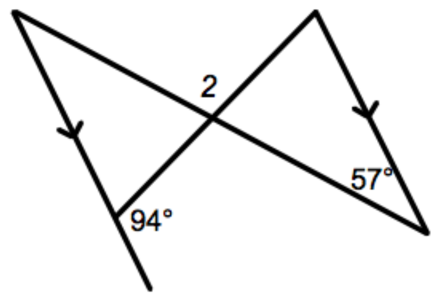
Exercises

Find the measure of the missing labeled angles.

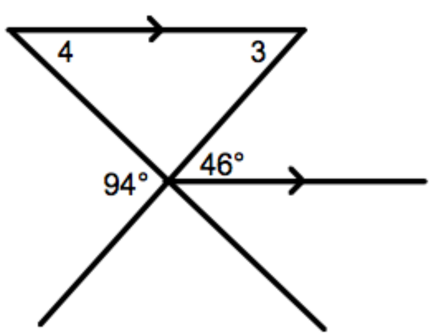
1.



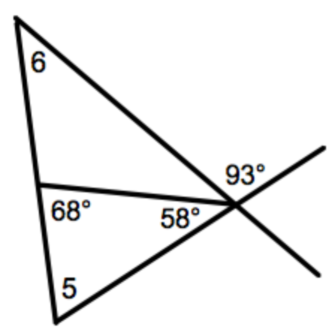
2.



3.



4.



Exercises

5. The degree measures of the angles of a triangle are represented by x, $3x$ and $5x - 54$. Find the value of x.

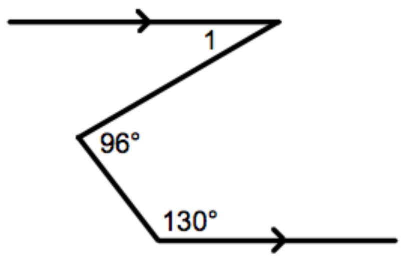
6. In ΔABC, the measure of $\angle B$ is 21 less than four times the measure of $\angle A$, and the measure of $\angle C$ is 1 more than five times the measure of $\angle A$. Find the measure, in degrees, of each angle of ΔABC.

7. In ΔABC, the measure of $\angle ABC$ is x^2, the measure of $\angle BCA$ is $-6x+100$, and the measure of $\angle CAB$ is $x+56$. Find the measure of $\angle ABC$.

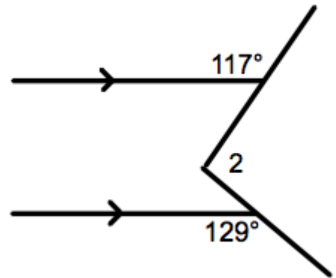
Homework

Find the measure of the missing labeled angles.

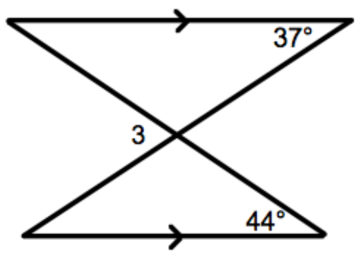
1.



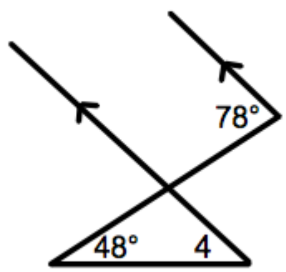
2.



3.



4.

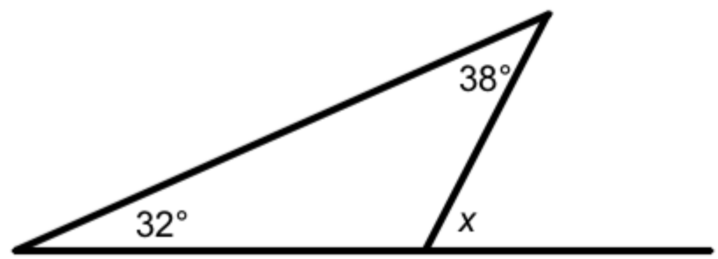


5. In ΔABC, the measure of $\angle A$ is 3 less than two times the measure of $\angle C$ and the measure of $\angle B$ is 11 more than the measure of $\angle C$. Find the measure of each angle of ΔABC.

Lesson 10: Solve for Unknown Angles – Angles in a Triangle

Opening Exercise

Find *x*:



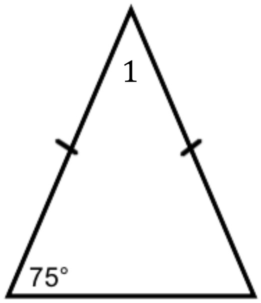
Fill in the "Fact/Discovery" column based on geometry facts you have learned in the past!

Name	Diagram	Fact/Discovery
∠ Sum of Right Δ		
Exterior ∠'s of a Δ		
Base ∠'s of Isosceles Δ		
Equilateral Triangle		

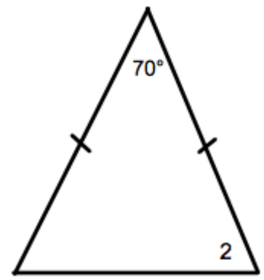
Exercises

In each figure, determine the measure of the unknown (labeled) angles. Show your work!!!

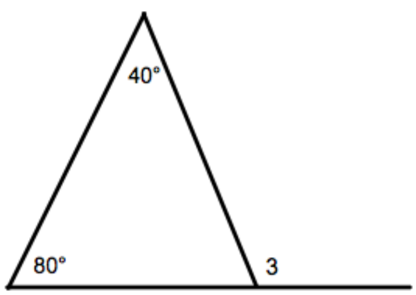
1.



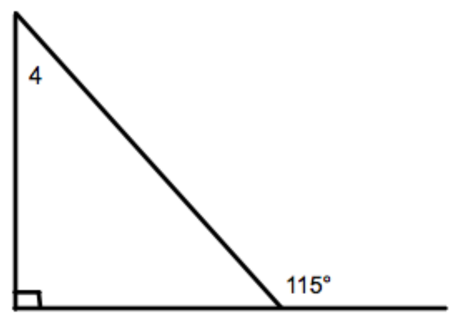
2.



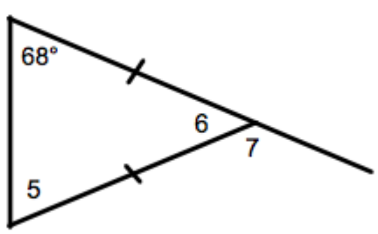
3.



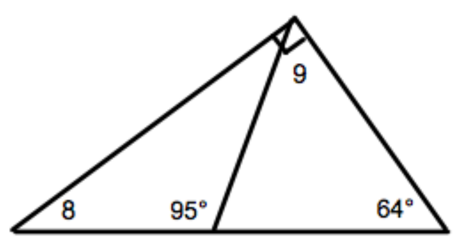
4.



5.



6.



Exercises

7. In ΔABC, the measure of angle *B* is three times as large as angle *A*. An exterior angle at *C* measures $140°$. Find the measure of angle *A*.

8. In ΔCAT, side $\overline{CT}$ is extended through *T* to *S*. If $\angle CAT = x^2 - 3x$, $\angle ACT = 6x + 20$, and $\angle ATS = 2x^2 - 5x$, find *x*.

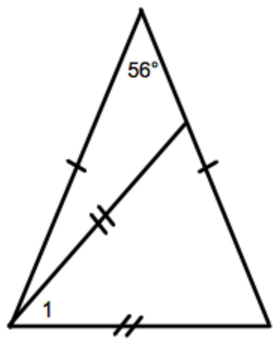
9. In isosceles triangle *ABC*, the vertex angle *C* is 20 more than twice the base angles. Find the measure of all the angles of this triangle.

10. In ΔDEF, $\angle D$ is a right angle and $\angle F$ is 12 degrees less than twice the measure of $\angle E$. Find $m\angle F$

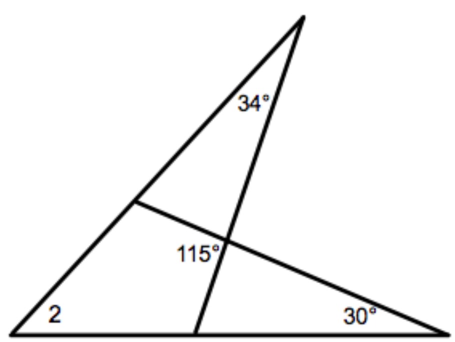
Exercises

A few more challenging questions! In each figure, determine the measure of the unknown (labeled) angles.

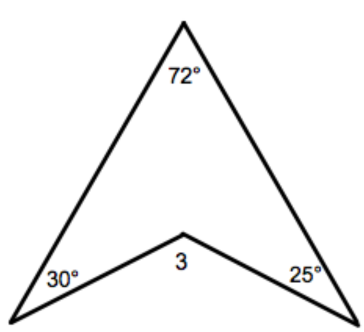
11.



12.



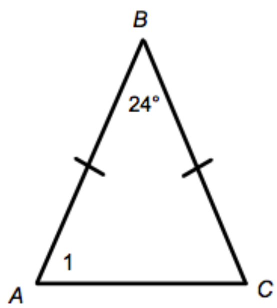
13.



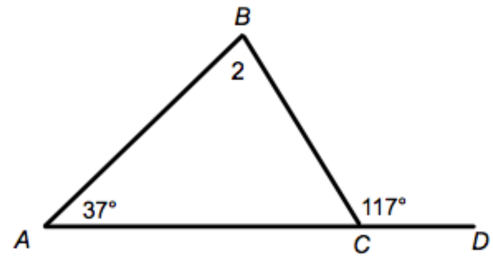
Homework

In questions 1 & 2, find the measure of the missing labeled angle.

1.

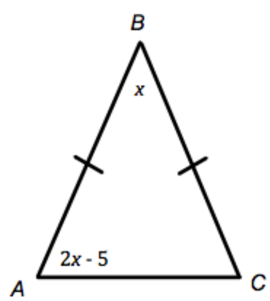


2.

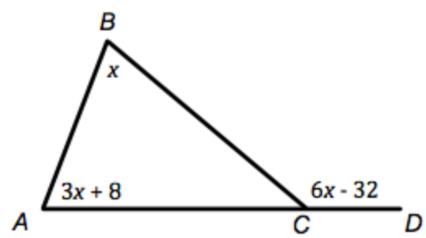


In questions 3 & 4, solve for *x*.

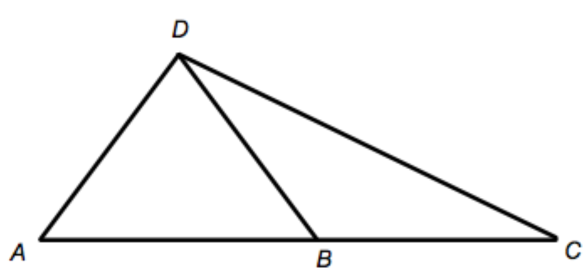
3.



4.



5. In the diagram below of ΔACD, B is a point on $\overline{AC}$ such that ΔADB is an equilateral triangle, and ΔDBC is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.



Lesson 11: Unknown Angle Proofs – Writing Proofs

Opening Exercise

Solve the following equation for *x*, showing every step in the solving process!!!

$$6x - 12 = 4x + 2$$

Now as a class we are going to solve this same problem as a formal proof.

Given: $6x - 12 = 4x + 2$
Prove: $x = 7$

Why Proofs?

One of the main goals in studying geometry is to develop your ability to reason critically: to draw valid conclusions based upon observations and proven facts. Master detectives do this sort of thing all the time. Take a look as Sherlock Holmes uses seemingly insignificant observations to draw amazing conclusions.

http://www.youtube.com/watch?v=o30UY_flFgM&feature=youtu.be

Could you follow Sherlock Holmes' reasoning as he described his thought process?

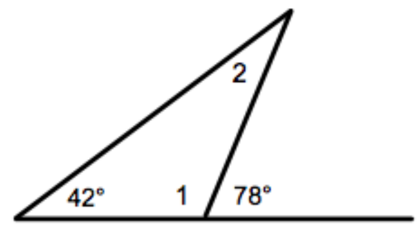
In a proof we are going to use known facts to end up with a newly proven fact. This is called **deductive reasoning**.

Example 1

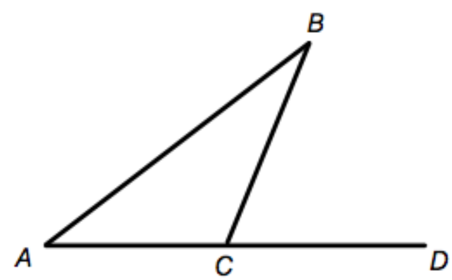
Given the diagram pictured to the right, find:

a. $m\angle 1$

b. $m\angle 2$



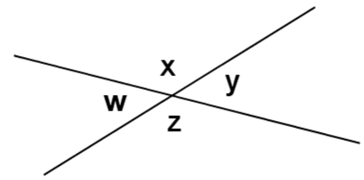
Now let's prove why the exterior angle of a triangle is equal to the sum of the remote interior angles!



Would this rule change if $\angle ACB$ was acute?

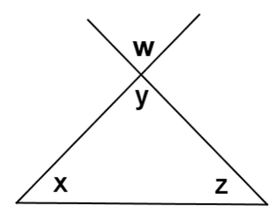
Exercises

1. Given the diagram pictured to the right, prove that vertical angles are congruent.



Statements	Reasons

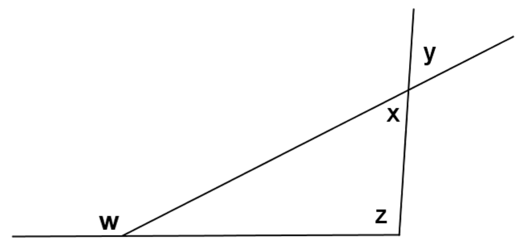
2. Given the diagram pictured to the right, prove that $\angle w + \angle x + \angle z = 180°$.



Statements	Reasons

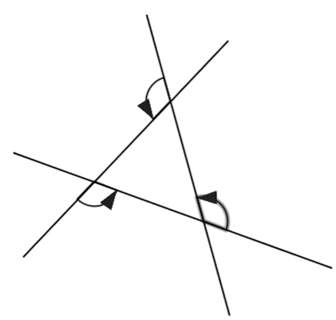
3. Given the diagram pictured to the right, prove that $\angle w = \angle y + \angle z$.

Statements	Reasons



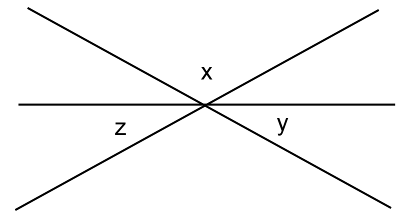
4. Given the diagram pictured to the right, prove that the sum of the angles marked by the arrows is $360°$.

Statements	Reasons



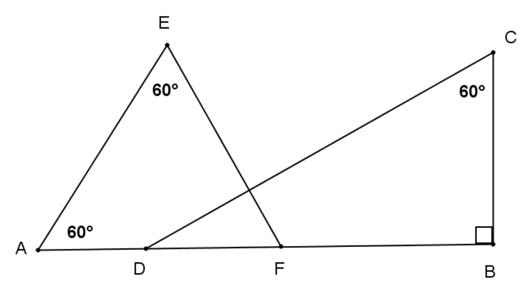
Homework

1. Given the diagram pictured to the right, prove that $\angle x + \angle y + \angle z = 180°$.



Statements	Reasons

2. Given the diagram pictured to the right, find the values of all the missing angles.

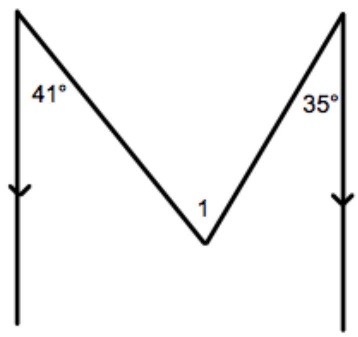


Lesson 12: Unknown Angle Proofs – Proofs with Constructions

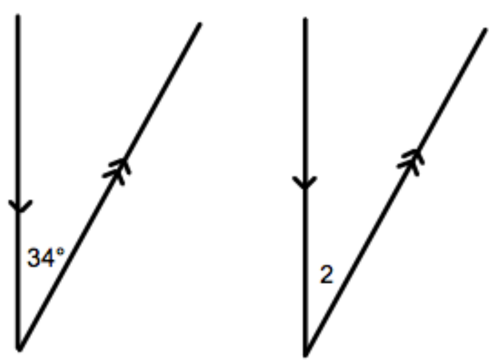
Opening Exercise

In each figure, determine the measure of the unknown (labeled) angles.

1.

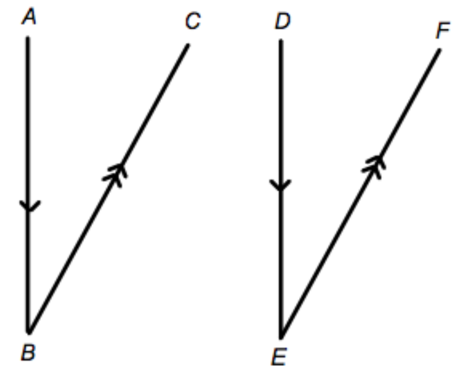


2.



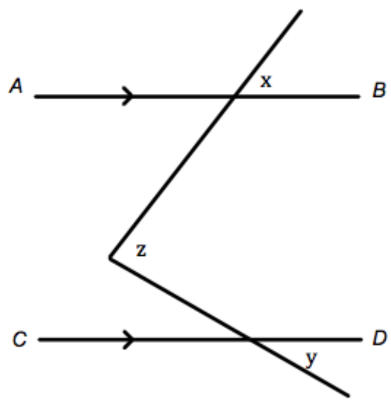
Example 1

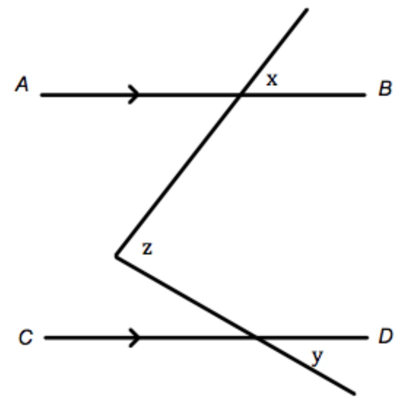
In the figure to the right, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$. Prove that $\angle B \cong \angle E$. (Hint: extend $\overline{BC}$ and $\overline{ED}$.)



Example 2

In the identical diagrams pictured below, there are at least 2 possibilities for auxiliary lines. Can you find them?

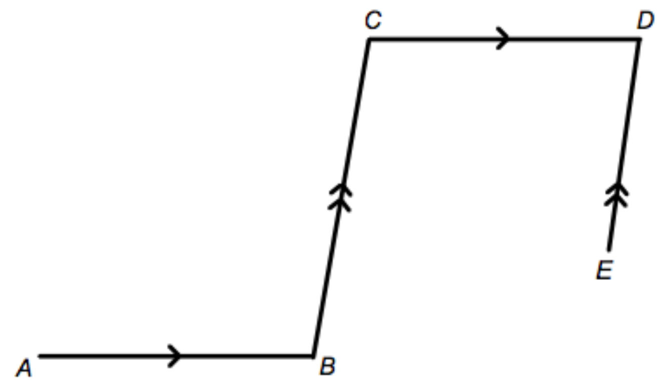




Given that $\overline{AB} \parallel \overline{CD}$, we are going to prove $z = x + y$. Half of the class is going to prove this using the first auxiliary line, while the other half is going to prove this using the second auxiliary line.

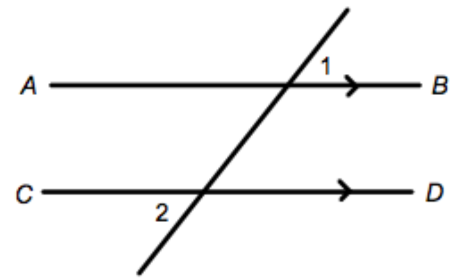
Example 3

In the figure to the right, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$.
Prove that $\angle B + \angle D = 180°$.

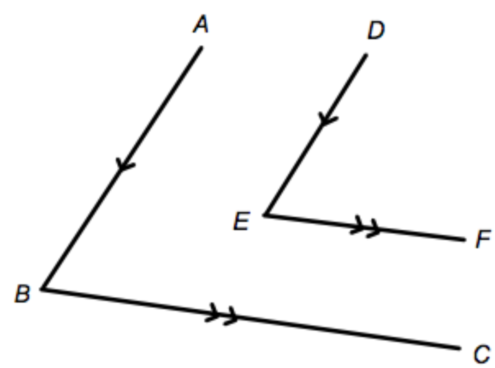


Homework

1. Prove the theorem "When parallel lines are cut by a transversal, alternate exterior angles are congruent". (This means you are proving $\angle 1 \cong \angle 2$ using other properties that you have learned.)



2. In the figure to the right, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$ Prove that $\angle ABC \cong \angle DEF$.



Lesson 13: Unknown Angle Proofs – Proofs of Known Facts

Opening Exercise

We proved *vertical angles are congruent* in Lesson 11 and we know that if a transversal intersects two parallel lines that the *alternate interior angles are congruent*. Using these known facts, prove the **corresponding angles are congruent**.

Vocabulary

- **Proof:** an explanation of how a mathematical statement follows logically from other known statements.

- **Theorem:** a mathematical statement that can be proven; typically written in the form "if (hypothesis) - then (conclusion)".

Exercise 1

Once a theorem has been proved, it can be added to our list of known fact and used in proofs of other theorems!

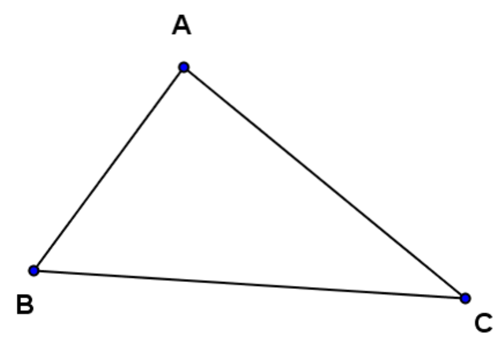
We now have available the following facts:

- Vertical angles are congruent. (vert. $\angle s$)
- Alternate interior angles are congruent. (alt. int. $\angle s$, $\overline{AB} \parallel \overline{CD}$)
- Corresponding angles are congruent. (corr. $\angle s$, $\overline{AB} \parallel \overline{CD}$)

We are going to prove one more in the homework:

- Interior angles on the same side of the transversal are supplementary. (int. $\angle s$, $\overline{AB} \parallel \overline{CD}$)

Use any of these four facts to prove that the three angles of a triangle sum to 180°. For this proof, you will need to draw an auxiliary line, parallel to one of the triangle's sides and passing through the vertex opposite that side. Add any necessary labels and write out your proof.

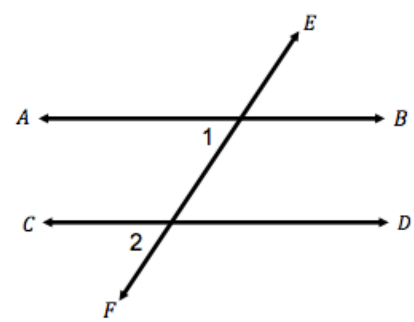


Exercise 2

Each of the three parallel line theorems has a converse (or reversing) theorem as follows:

Original	Converse
If two parallel lines are cut by a transversal, then alternate interior angles are congruent.	If two lines are cut by a transversal such that alternate interior angles are congruent, then the lines are parallel.
If two parallel lines are cut by a transversal, then corresponding angles are congruent.	If two lines are cut by a transversal such that corresponding angles are congruent, then the lines are parallel.
If two parallel lines are cut by a transversal, then interior angles on the same side of the transversal add to 180°.	If two lines are cut by a transversal such that interior angles on the same side of the transversal add to 180°, then the lines are parallel.

In the figure at the right, $\angle 1 \cong \angle 2$.
Prove that $\overline{AB} \parallel \overline{CD}$.



Exercise 3

Construct a proof designed to demonstrate the following:

If two lines are perpendicular to the same line, they are parallel to each other.

(a) Draw and label a diagram.
(b) State the given facts and the conjecture to be proved.
(c) Write out a clear statement of your reasoning to justify each step.

Homework

1. Prove: *When parallel lines are cut by a transversal, the same side interior angles are supplementary.*

2. Given: Quadrilateral $ABCD$
$\angle C$ and $\angle D$ are supplementary.
$\angle B \cong \angle D$
Prove: $\overline{AB} \parallel \overline{CD}$