Unit 3
Congruence & Proofs

Lesson 1: Introduction to Triangle Proofs

Opening Exercise

Using your knowledge of angle and segment relationships from Unit 1, fill in the following:

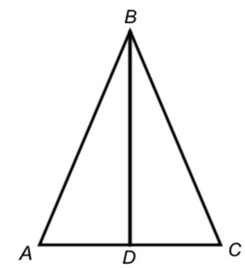
Definition/Property/Theorem	**Diagram/Key Words**	**Statement**
Definition of **Right Angle**		
Definition of **Angle Bisector**		
Definition of **Segment Bisector**		
Definition of **Perpendicular**		
Definition of **Midpoint**		
Angles on a Line		
Angles at a Point		
Angles Sum of a Triangle		
Vertical Angles		

Example 1

We are now going to take this knowledge and see how we can apply it to a proof. In each of the following you are given information. You must interpret what this means by first marking the diagram and then writing it in proof form.

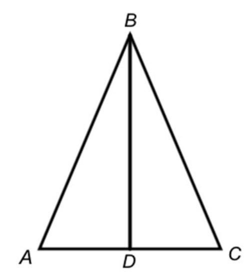
a. Given: D is the midpoint of $\overline{AC}$

Statements	Reasons
1. D is the midpoint of $\overline{AC}$	1. Given
2.	2.



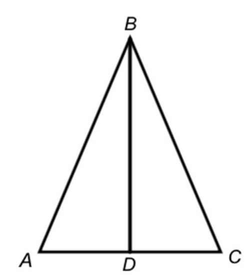
b. Given: $\overline{BD}$ bisects $\overline{AC}$

Statements	Reasons
1. $\overline{BD}$ bisects $\overline{AC}$	1. Given
2.	2.



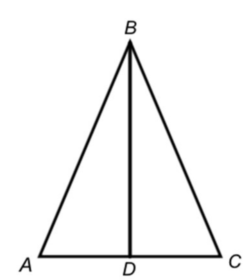
c. Given: $\overline{BD}$ bisects $\angle ABC$

Statements	Reasons
1. $\overline{BD}$ bisects $\angle ABC$	1. Given
2.	2.



d. Given: $\overline{BD} \perp \overline{AC}$

Statements	Reasons
1. $\overline{BD} \perp \overline{AC}$	1. Given
2.	2.
3.	3.



Example 2

Listed below are other useful properties we've discussed that will be used in proofs.

Property / Postulate	In Words	Statement
Addition Postulate	Equals added to equals are equal.	
Subtraction Postulate	Equals subtracted from equals are equal.	
Multiplication Postulate	Equals multiplied by equals are equal.	
Division Postulate	Equals divided by equals are equal.	
Partition Postulate	The whole is equal To the sum of its parts.	
Substitution	A quantity may be substituted for an equal quantity.	
Reflexive	Anything is equal to itself	

<u>The two most important properties about parallel lines to remember</u>:

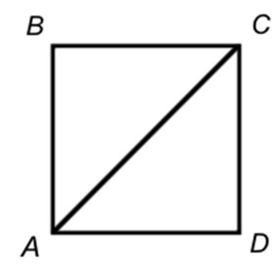
1.

2.

Homework

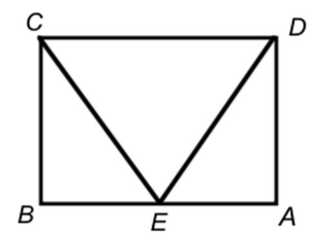
Given the following information, mark the diagram and then state your markings in proof form.

1. Given: $\overline{AC}$ bisects $\angle BCD$



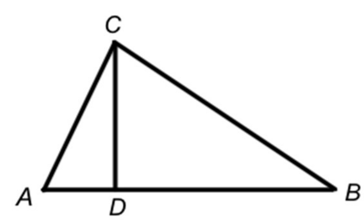
Statements	Reasons
1. $\overline{AC}$ bisects $\angle BCD$	1.Given
2.	2.

2. Given: E is the midpoint of $\overline{AB}$



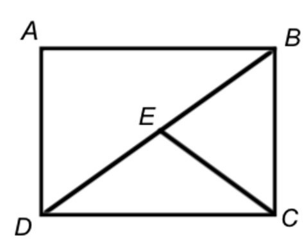
Statements	Reasons
1. E is the midpoint of $\overline{AB}$	1.Given
2.	2.

3. Given: $\overline{CD} \perp \overline{AB}$



Statements	Reasons
1. $\overline{CD} \perp \overline{AB}$	1.Given
2.	2.
3.	3.

4. Given: $\overline{CE}$ bisects $\overline{BD}$



Statements	Reasons
1. $\overline{CE}$ bisects $\overline{BD}$	1.Given
2.	2.

Lesson 2: Congruence Criteria for Triangles - SAS

Opening Exercise

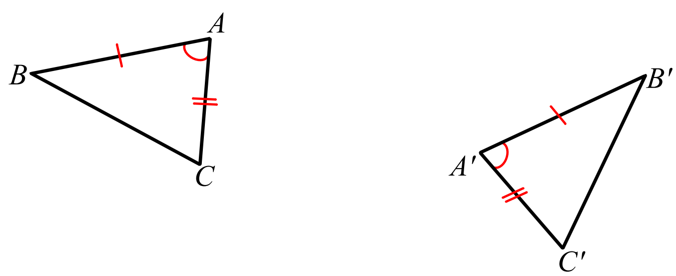
In Unit 2 we defined **congruent** to mean there exists a composition of basic rigid motions of the plane that maps one figure to the other.

In order to prove triangles are congruent, we do *not* need to prove all of their corresponding parts are congruent. Instead we will look at criteria that refer to fewer parts that will guarantee congruence.

We will start with:

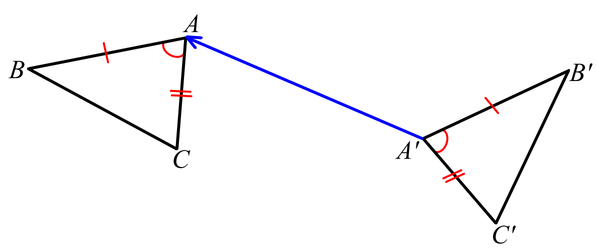
Side-Angle-Side Triangle Congruence Criteria (SAS)

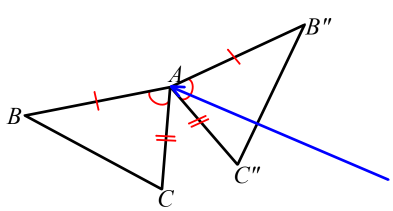
- Two pairs of sides and the included angle are congruent



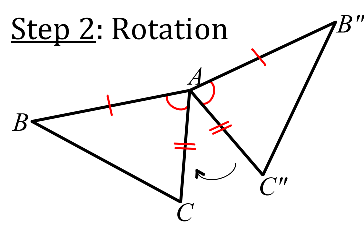
Using these distinct triangles, we can see there is a composition of rigid motions that will map $\Delta A'B'C'$ to ΔABC.

Step 1: Translation

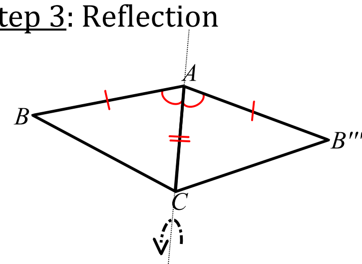




Step 2: Rotation

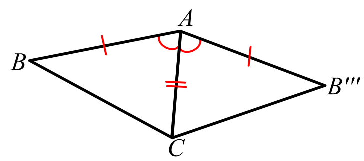
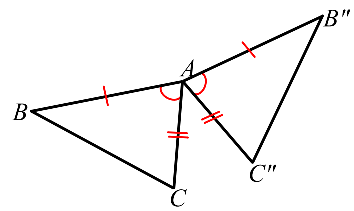


Step 3: Reflection



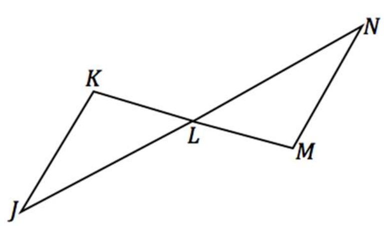
Example 1

What if we had SAS criteria for two triangles that were not distinct? Consider the following two cases and determine the rigid motion(s) that are needed to demonstrate congruence.

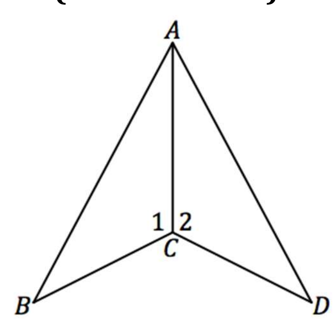
Case	Diagram	Rigid Motion(s) Needed
Shared Side		
Shared Vertex		

<u>Two properties to look for when doing triangle proofs</u>:

Vertical Angles

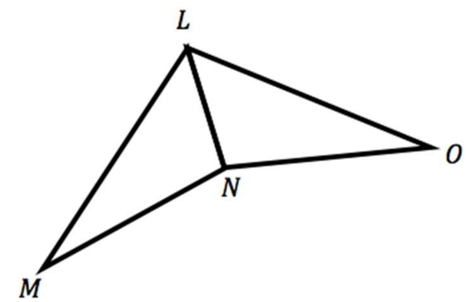


Reflexive Property (Common Side)



Examples

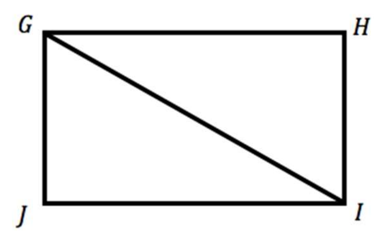
2. Given: $\angle LNM \cong \angle LNO$, $\overline{MN} \cong \overline{ON}$



a. Prove: $\Delta LMN \cong \Delta LON$

b. Describe the rigid motion(s) that would map ΔLON onto ΔLMN.

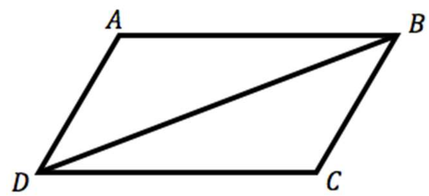
3. Given: $\angle HGI \cong \angle JIG$, $HG \cong JI$



a. Prove: $\Delta HGI \cong \Delta JIG$

b. Describe the rigid motion(s) that would map ΔJIG onto ΔHGI.

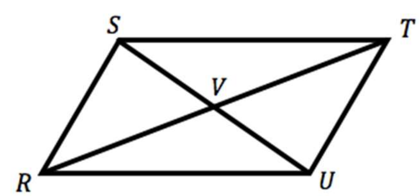
4. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$



a. Prove: $\Delta ABD \cong \Delta CDB$

b. Describe the rigid motion(s) that would map ΔCDB onto ΔABD.

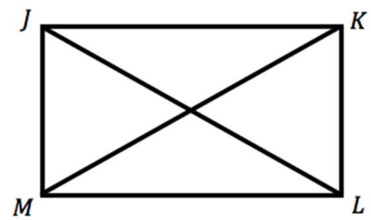
5. Given: $\overline{SU}$ and $\overline{RT}$ bisect each other



a. Prove: $\Delta SVR \cong \Delta UVT$

b. Describe the rigid motion(s) that would map ΔUVT onto ΔSVR.

6. Given: $\overline{JM} \cong \overline{KL}, \overline{JM} \perp \overline{ML}, \overline{KL} \perp \overline{ML}$



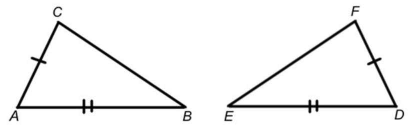
a. Prove: $\Delta JML \cong \Delta KLM$

b. Describe the rigid motion(s) that would map ΔJML onto ΔKLM.

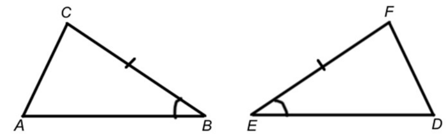
Homework

1. In order to use SAS to prove the following triangles congruent, draw in the missing labels:

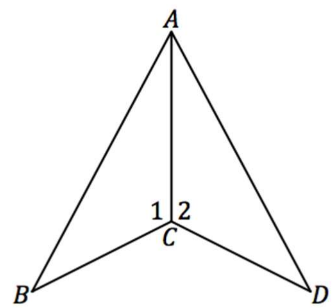
a



b.



2. <u>Given</u>: $\angle 1 \cong \angle 2$, $\overline{BC} \cong \overline{DC}$

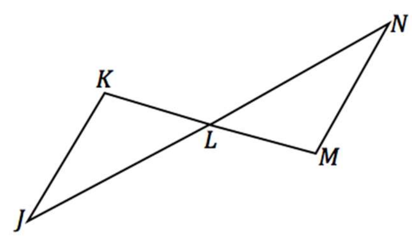


a. <u>Prove</u>: $\Delta ABC \cong \Delta ADC$

b. Describe the rigid motion(s) that would map ΔADC onto ΔABC.

3. <u>Given</u>: *KM* and *JN* bisect each other

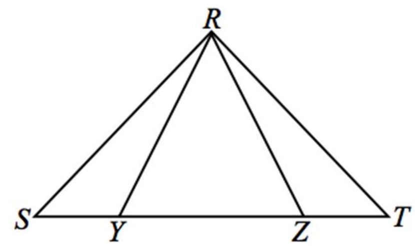
a. <u>Prove</u>: $\Delta JKL \cong \Delta NML$



b. Describe the rigid motion(s) that would map ΔNML onto ΔJKL.

Lesson 3: Base Angles of Isosceles Triangles

Opening Exercise



Given: ΔRST is isosceles with $\angle R$ as the vertex,
$\overline{SY} \cong \overline{TZ}$

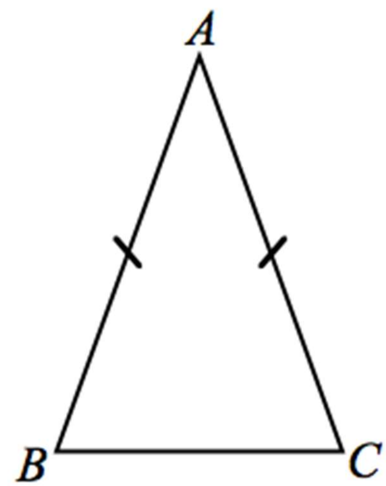
Prove: $\Delta RSY \cong \Delta RTZ$

Example 1

You will need a compass and a straightedge

We are going to prove that the base angles of an isosceles triangle are congruent!

Given: Isosceles ΔABC with $\overline{AB} \cong \overline{AC}$
Goal: To prove $\angle B \cong \angle C$



Step 1: Construct the angle bisector of the vertex $\angle$.

Step 2: ΔABC has now been split into two triangles.
Prove the two triangles are $\cong$.

Step 3: Identify the corresponding sides and angles.

Step 4: What is true about $\angle B$ and $\angle C$?

Step 5: What types of angles were formed when the angle bisector intersected $\overline{BC}$?
What does this mean about the angle bisector?

Once we prove triangles are congruent, we know that their corresponding parts (angles and sides) are congruent. We can abbreviate this is in a proof by using the reasoning of:

CPCTC (Corresponding Parts of Congruent Triangles are Congruent).

<u>To Prove Angles or Sides Congruent</u>:

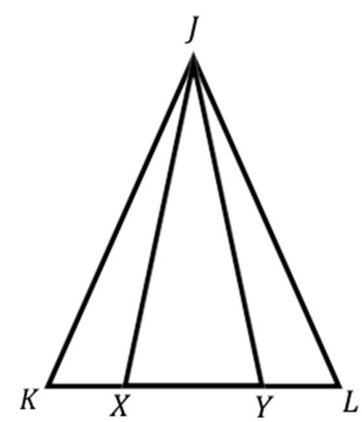
1. Prove the triangles are congruent (using one of the above criteria)
2. States that the angles/sides are congruent because of CPCTC.

<u>To Prove Midpoint/Bisect/Perpendicular/Parallel</u>:

1. Prove the triangles are congruent (using one of the above criteria)
2. State that the necessary angles/sides are congruent because of CPCTC.
3. State what you are trying to prove using def. of midpoint/def. of bisect/etc.

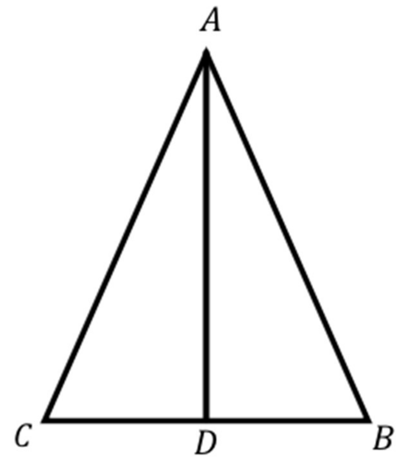
Example 2

<u>Given</u>: ΔJKL is isosceles, $\overline{KX} \cong \overline{LY}$
<u>Prove</u>: ΔJXY is isosceles

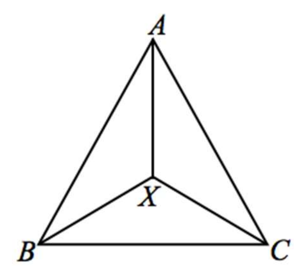


Homework

1. Given: Isosceles ΔABC with $\angle A$ as the vertex angle
D is the midpoint of $\overline{BC}$
Prove: $\Delta ACD \cong \Delta ABD$



2. Given: $\overline{BA} \cong \overline{CA}$, $\overline{AX}$ is the angle bisector of $\angle BAC$
Prove: $\angle ABX \cong \angle ACX$



Lesson 4: Congruence Criteria for Triangles – ASA and SSS

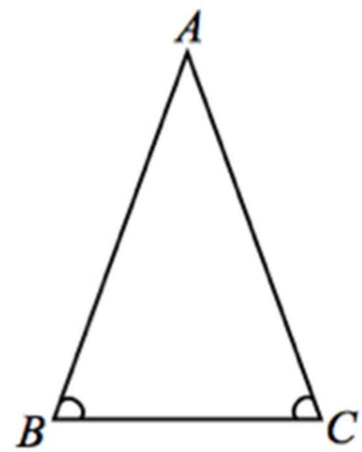
Opening Exercise

You will need a compass and a straightedge

1. Given: $\triangle ABC$ with $\angle B \cong \angle C$
   Goal: To prove $\overline{BA} \cong \overline{CA}$

   Step 1: Construct the perpendicular bisector to $\overline{BC}$.

   Step 2: $\triangle ABC$ has now been split into two triangles.
   Prove $\overline{BA} \cong \overline{CA}$.



2. In the diagram, $\triangle ABC$ is isosceles with $\overline{AC} \cong \overline{AB}$. In your own words, describe how the properties of rigid motions can be used to show $\angle B \cong \angle C$.

There are 5 ways to test for triangle congruence.

In lesson 1 we saw that we can prove triangles congruent using **SAS**. We proved this using rigid motions. Here's another way to look at it:

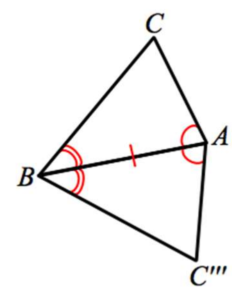
http://www.mathopenref.com/congruentsas.html

Today we are going to focus on two more types:

Angle-Side-Angle Triangle Congruence Criteria (ASA)

- Two pairs of angles and the included side are congruent

To prove this we could start with two distinct triangles. We could then translate and rotate one to bring the congruent sides together like we did in the SAS proof (see picture to the right).

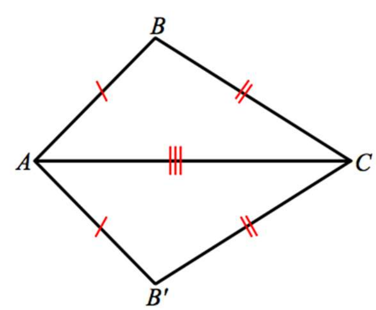


As we can see, a reflection over *AB* would result in the triangles being mapped onto one another, producing two congruent triangles.

http://www.mathopenref.com/congruentasa.html

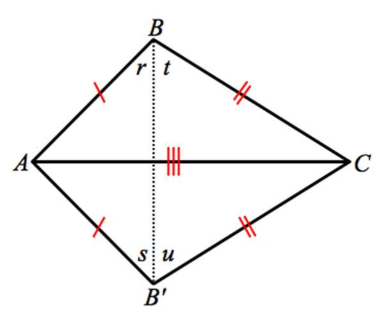
Side-Side-Side Triangle Congruence Criteria (SSS)

- All of the corresponding sides are congruent



Without any information about the angles, we cannot just perform a reflection as we did in the other two proofs. But by drawing an auxiliary line, we can see that two isosceles triangles are formed, creating congruent base angles and therefore, $\angle B \cong \angle B'$.

We can now perform a reflection, producing two congruent triangles.

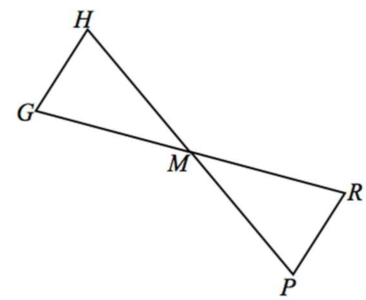


http://www.mathopenref.com/congruentsss.html

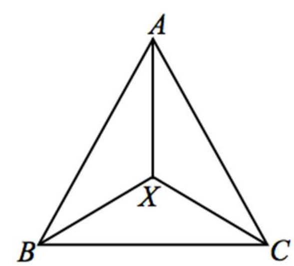
Exercises

Prove the following using any method of triangle congruence that we have discussed. Then identify the rigid motion(s) that would map one triangle onto the other.

1. Given: M is the midpoint of $\overline{HP}$, $\angle H \cong \angle P$
   Prove: $\Delta GHM \cong \Delta RPM$

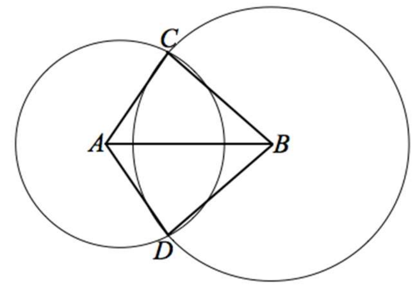


2. Given: $\overline{AB} \cong \overline{AC}$, $\overline{XB} \cong \overline{XC}$
   Prove: $\overline{AX}$ bisects $\angle BAC$



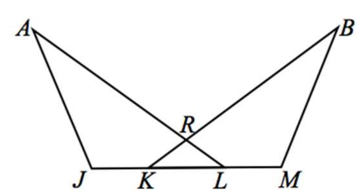
Example 1

Given: Circles with centers A and B intersect at C and D.
Prove: $\angle CAB \cong \angle DAB$



Example 2

Given: $\angle J \cong \angle M$, $\overline{JA} \cong \overline{MB}$, $\overline{JK} \cong \overline{ML}$
Prove: $\overline{KR} \cong \overline{LR}$

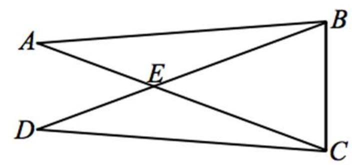


Homework

Prove the following using any method of triangle congruence that we have discussed. Then identify the rigid motion(s) that would map one triangle onto the other.

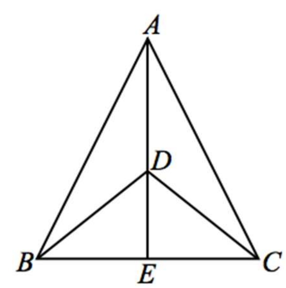
1. Given: $\angle A \cong \angle D$, $\overline{AE} \cong \overline{DE}$

   Prove: $\Delta AEB \cong \Delta DEC$



2. Given: $\overline{BD} \cong \overline{CD}$, E is the midpoint of $\overline{BC}$

   Prove: $\angle AEB \cong \angle AEC$

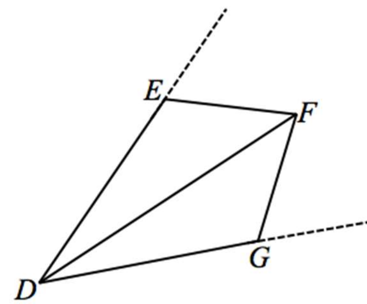


Lesson 5: Congruence Criteria for Triangles – AAS and HL

Opening Exercise

Write a proof for the following question. When finished, compare your proof with your partner's.

Given: $\overline{DE} \cong \overline{DG}$, $\overline{EF} \cong \overline{GF}$
Prove: $\overline{DF}$ is the angle bisector of $\angle EDG$



We have now identified 3 different ways of proving triangles congruent. What are they?

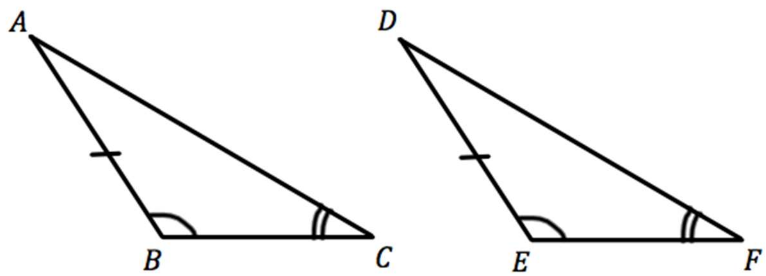
Does this mean any combination of 3 pairs of congruent sides and/or angles will guarantee congruence?

Let's try another combination of sides and angles:

Angle-Angle-Side Triangle Congruence Criteria (AAS)

- Two pairs of angles and a side that is not included are congruent

To prove this we could start with two distinct triangles.



If $\angle B \cong \angle E$ and $\angle C \cong \angle F$, what must be true about $\angle A$ and $\angle D$? Why?

Therefore, **AAS** is actually an extension of which triangle congruence criterion?

Let’s take a look at two more types of criteria:

Angle-Angle-Angle (AAA)

- All three pairs of angles are congruent

  http://www.mathopenref.com/congruentaaa.html

Does **AAA** guarantee triangle congruence? Draw a sketch demonstrating this.

Side-Side-Angle (SSA)

- Two pairs of sides and a non-included angle are congruent

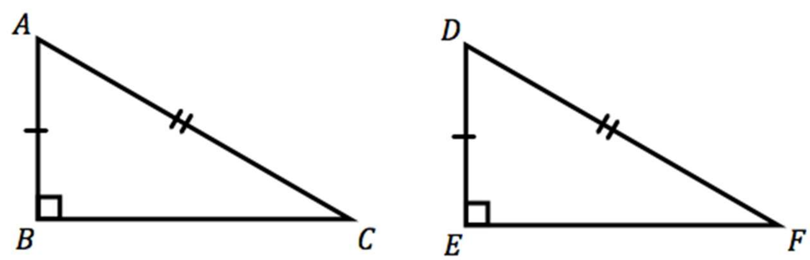
  http://www.mathopenref.com/congruentssa.html

Does **SSA** guarantee triangle congruence? Draw a sketch demonstrating this.

There is a special case of **SSA** that does work, and that is when dealing with right triangles. We call this **Hypotenuse-Leg** triangle congruence.

Hypotenuse-Leg Triangle Congruence Criteria(HL)

- When two right triangles have congruent hypotenuses and a pair of congruent legs, then the triangles are congruent.



If we know two sides of a right triangle, how could we find the third side?

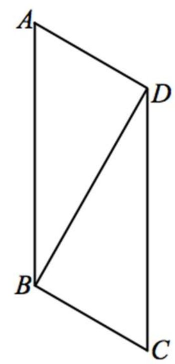
Therefore, **HL** is actually an extension of which triangle congruence criterion?

In order to use **HL** triangle congruence, you must first state that the triangles are right triangles!

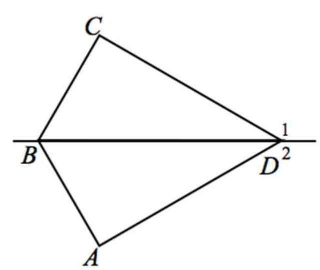
Exercises

Prove the following using any method of triangle congruence that we have discussed. Then identify the rigid motion(s) that would map one triangle onto the other.

1. Given: $\overline{AD} \perp \overline{BD}$, $\overline{BD} \perp \overline{BC}$, $\overline{AB} \cong \overline{CD}$
   Prove: $\Delta ABD \cong \Delta CDB$



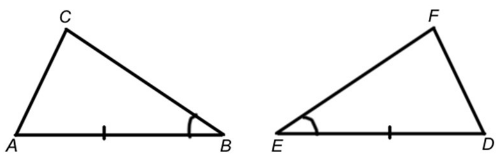
2. Given: $\overline{BC} \perp \overline{CD}$, $\overline{AB} \perp \overline{AD}$, $\angle 1 \cong \angle 2$
   Prove: $\Delta BCD \cong \Delta BAD$



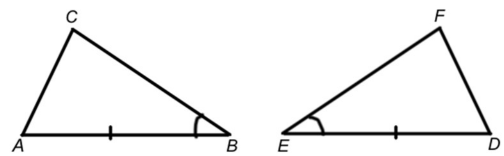
Homework

In 1-4, mark the appropriate congruence markings to use the method of proving that is stated:

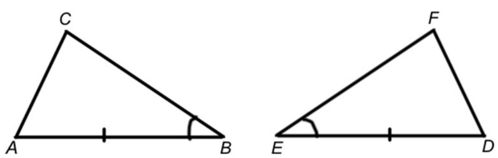
1. SAS



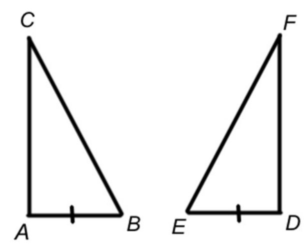
2. AAS



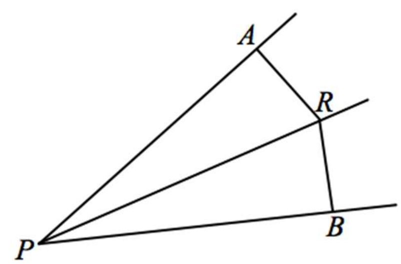
3. ASA



4. HL



5. <u>Given</u>: $\overline{PA} \perp \overline{AR}$, $\overline{PB} \perp \overline{BR}$, $\overline{AR} \cong \overline{BR}$
<u>Prove</u>: PR bisects $\angle APB$



Lesson 6: Triangle Congruency Proofs

Opening Exercise

Triangle proofs summary. Let's see what you know!

List the 5 ways of proving triangles congruent:

1.
2.
3.
4.
5.

What two sets of criteria **CANNOT** be used to prove triangles congruent:

1.
2.

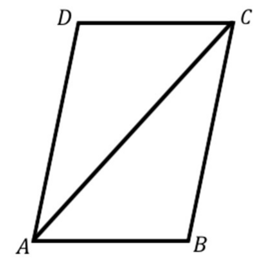
In order to prove a pair of corresponding sides or angles are congruent, what must you do first?

What is the abbreviation used to state that corresponding parts (sides or angles) of congruent triangles are congruent?

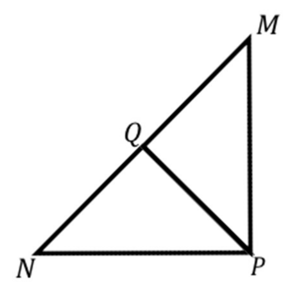
Exercises

Prove the following using any method of triangle congruence that we have discussed.

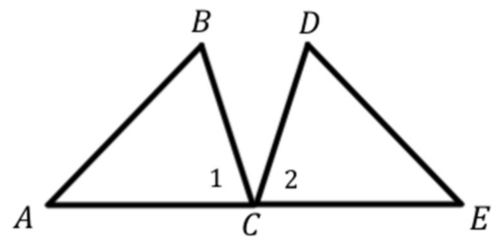
1. Given: $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{DA}$
Prove: $\Delta ADC \cong \Delta CBA$



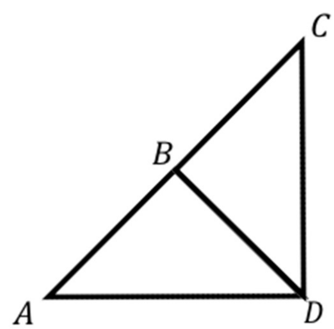
2. Given: $\overline{NQ} \cong \overline{MQ}$
$\overline{PQ} \perp \overline{NM}$
Prove: $\Delta PQN \cong \Delta PQM$



3. Given: $\angle 1 \cong \angle 2$, $\angle A \cong \angle E$,
C is the midpoint of $\overline{AE}$
Prove: $\overline{BC} \cong \overline{DC}$



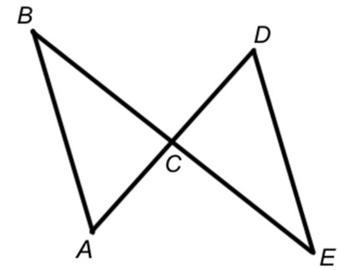
4. Given: $\overline{BD}$ bisects $\angle ADC$
$\angle A \cong \angle C$
Prove: $\overline{AB} \cong \overline{CB}$



5. Given: $\overline{AD}$ bisects $\overline{BE}$

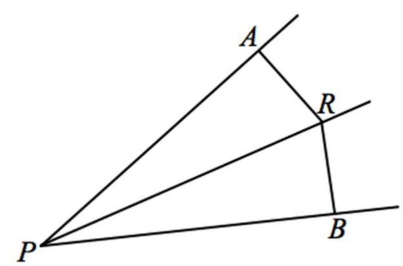
$AB \parallel DE$

Prove: $\Delta ABC \cong \Delta DEC$



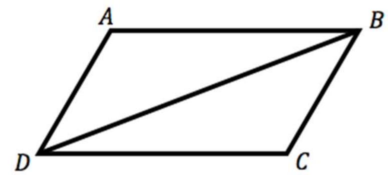
6. Given: $PA \perp AR$, $PB \perp BR$, $\overline{AR} \cong \overline{BR}$

Prove: PR bisects $\angle APB$

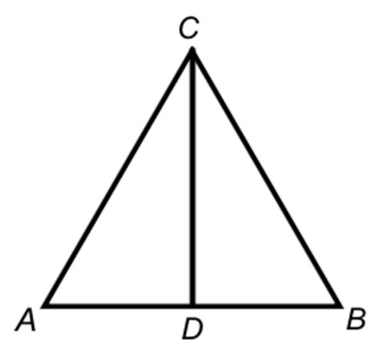


Homework

1. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$
Prove: $\Delta ABD \cong \Delta CDB$



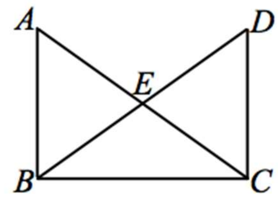
2. Given: $\overline{CD} \perp \overline{AB}$, $\overline{CD}$ bisects $\overline{AB}$, $\overline{AC} \cong \overline{BC}$
Prove: $\Delta ACD \cong \Delta BCD$



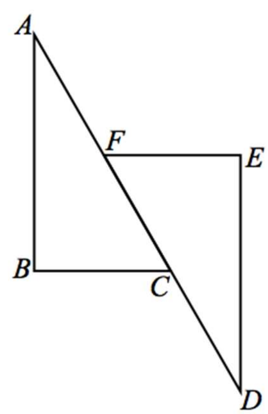
Lesson 7: Triangle Congruency Proofs II

Prove the following using any method of triangle congruence that we have discussed.

1. Given: $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{DC}$
   $\overline{DB}$ bisects $\angle ABC$
   $\overline{AC}$ bisects $\angle DCB$
   $\overline{EB} \cong \overline{EC}$
   Prove: $\Delta BEA \cong \Delta CED$



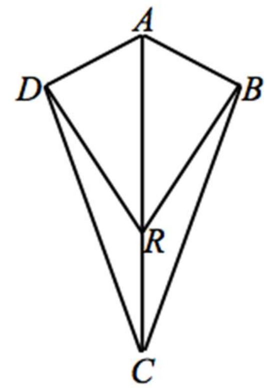
2. Given: $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$, $\overline{AF} \cong \overline{DC}$
   Prove: $\Delta ABC \cong \Delta DEF$



3. Given: $\overline{AD} \perp \overline{DR}$, $\overline{AB} \perp \overline{BR}$

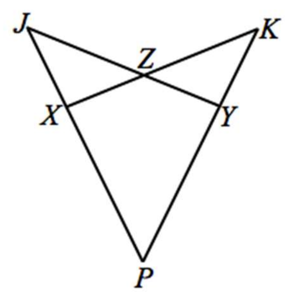
$\overline{AD} \cong \overline{AB}$

Prove: $\angle ARD \cong \angle ARB$



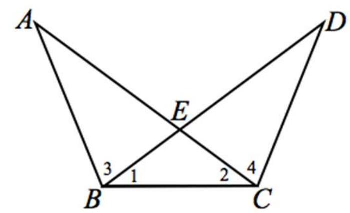
4. Given: $\overline{XJ} \cong \overline{YK}$, $\overline{PX} \cong \overline{PY}$, $\angle ZXJ \cong \angle ZYK$

Prove: $\overline{JY} \cong \overline{KX}$



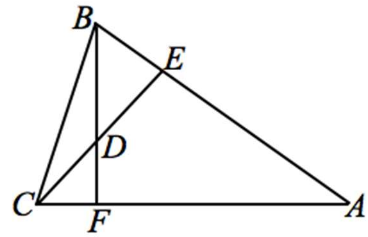
5. <u>Given</u>: $\angle 1 \cong \angle 2, \ \angle 3 \cong \angle 4$

<u>Prove</u>: $\overline{AC} \cong \overline{BD}$

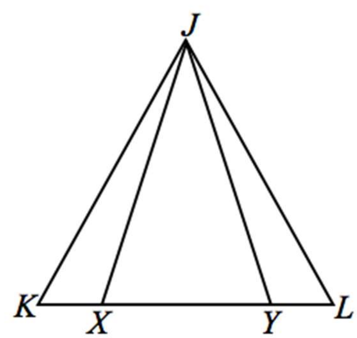


Homework

1. Given: $\overline{BF} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$
$\overline{AE} \cong \overline{AF}$
Prove: $\Delta ACE \cong \Delta ABF$



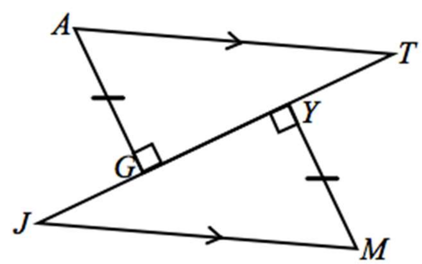
2. Given: $\overline{JK} \cong \overline{JL}$, $\overline{JX} \cong \overline{JY}$
Prove: $\overline{KX} \cong \overline{LY}$



Lesson 8: Properties of Parallelograms

Opening Exercise

Based on the diagram pictured below, answer the following:



1. If the triangles are congruent, state the congruence.

2. Which triangle congruence criterion guarantees they are congruent?

3. Side TG corresponds with which side of ΔMYJ?

Vocabulary

Define	Diagram
Parallelogram	

Using this definition of parallelograms and our knowledge of triangle congruence, we can prove the following properties of parallelograms:

- Opposite sides are congruent
- Opposite angles are congruent
- Diagonals bisect each other
- One pair of opposite sides are parallel and congruent

Example 1

We are going to prove the following sentence:

If a quadrilateral is a parallelogram, then its opposite sides and angles are equal in measure.

Given:

Diagram:

Prove:

Proof:

Example 2

Now that we have proven that opposite sides and angles of a parallelogram are congruent, we can use that it on our proofs!

We are going to prove the following sentence:

If a quadrilateral is a parallelogram, then the diagonals bisect each other.

Given:

Diagram:

Prove:

Proof:

Example 3

We are going to prove the following sentence:

If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Given:

Diagram:

Prove:

Proof:

Homework

Prove the following sentence:

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Given:

Diagram:

Prove:

Proof:

Lesson 9: Properties of Parallelograms II

Opening Exercise

Draw a diagram for each of the quadrilaterals listed and draw in congruence markings where you believe they exist.

Parallelogram

Rhombus

Rectangle

Square

Family of Quadrilaterals

Many of the quadrilaterals listed in the Opening Exercise share some of the same properties. We can look at this as a family:

The quadrilaterals at the bottom have all of the properties of the figures listed above it. Based on this, determine if the following are true or false. If it is false, explain why.

1. All rectangles are parallelograms.

2. All parallelograms are rectangles.

3. All squares are rectangles.

4. All rectangles are squares.

	Parallelogram	Rhombus	Rectangle	Square	Trapezoid
2 Pairs opp. Sides parallel					
Opposite sides ≅					
Opposite angles ≅					
Consecutive angles supplementary					
Diagonals bisect each other					
4 ≅ sides					
4 ≅angles					
Diagonals perpendicular					
Diagonals bisect opp. Angles					
Diagonals ≅					
One pair of parallel sides					

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Use to show if sides or diagonals are congruent

Slope Formula: $m = \frac{vertical\ change}{horizontal\ change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Used to show if sides parallel or perpendicular

(same slope = parallel   opposite reciprocal slopes = perpendicular)

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Use to show if the diagonals bisect each other

(same midpoint means diagonals bisect each other)

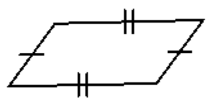
Algebra *HIJK* is a rectangle. Find the value of *x* and the length of each diagonal.

1. $HJ = 4x$ and $IK = 7x - 12$

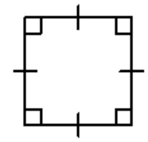
2. $HJ = x + 40$ and $IK = 5x$

Determine the most precise name for each quadrilateral.

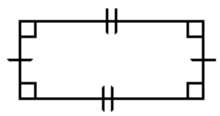
3



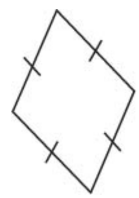
4.



5.

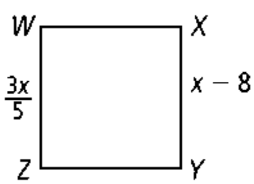


6.

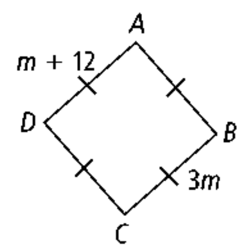


Algebra Find the values of the variables. Then find the side lengths.

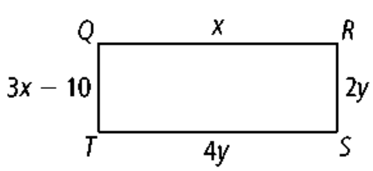
7. square *WXYZ*



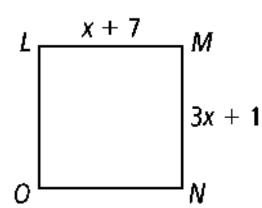
8. rhombus *ABCD*



9. rectangle *QRST*



10. square *LMNO*



11. Solve using a paragraph proof.

Given: Rectangle *DVEO* with diagonals $\overline{DE}$ and $\overline{OV}$

Prove: $\triangle OVE \cong \triangle DEV$

Example 1

Prove the following sentence:

If a parallelogram is a rectangle, then the diagonals are equal in length.

Given:

Diagram:

Prove:

Proof:

Example 2

Prove the following sentence:

If a parallelogram is a rhombus, the diagonals intersect perpendicularly.

<u>Given</u>:

<u>Diagram</u>:

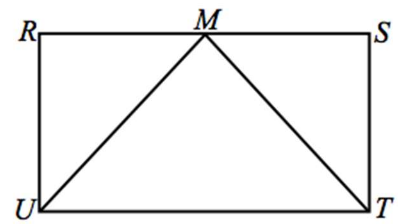
<u>Prove</u>:

<u>Proof</u>:

Homework

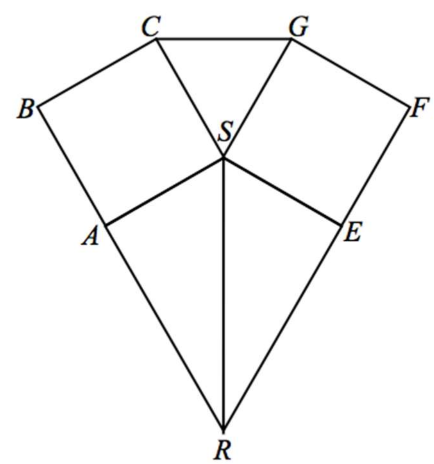
1. Given: Rectangle *RSTU*, *M* is the midpoint of *RS*

   Prove: ΔUMT is isosceles



2. Given: Square *ABCS* ≅ Square EFGS

   Prove: $\Delta ASR \cong \Delta ESR$

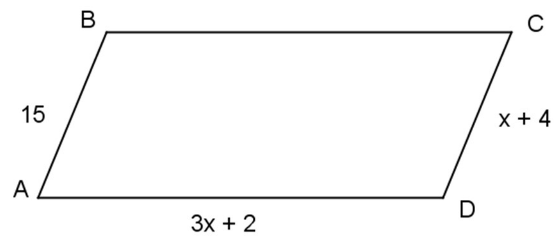


Lesson 10: Mid-segment of a Triangle

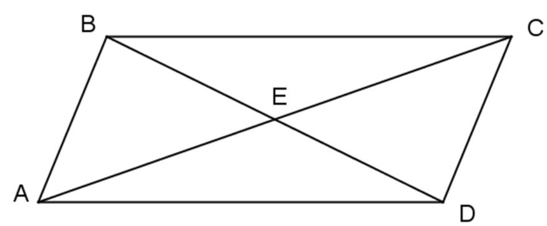
Opening Exercise

Using your knowledge of the properties of parallelograms, answer the following questions:

1. Find the perimeter of parallelogram *ABCD*. Justify your solution.



2. If $AC = 34$, $AB = 26$ and $BD = 28$, find the perimeter of ΔCED. Justify your solution.



Vocabulary

Define	Diagram
Mid-segment	

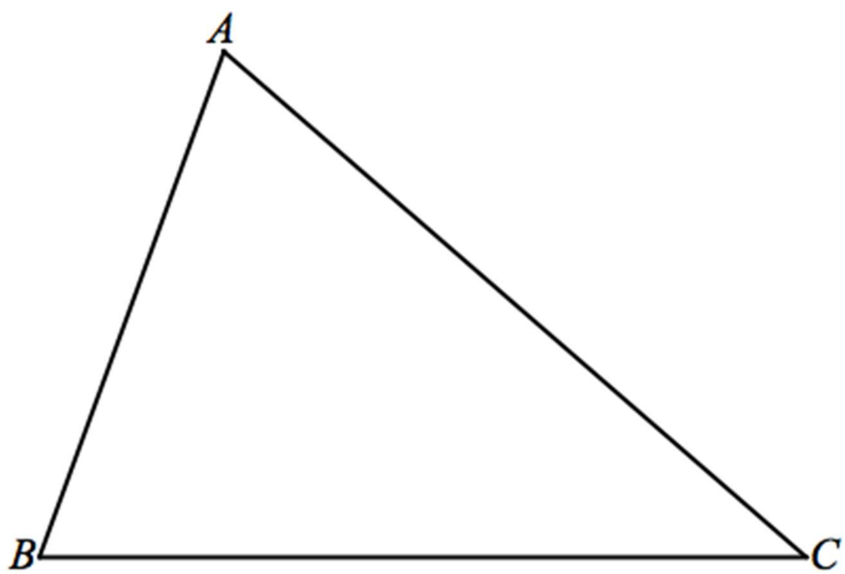
Example 1

You will need a compass and a straightedge

We are going to construct a mid-segment.

Steps:

1. Construct the midpoints of AB and AC and label them as X and Y, respectively.
2. Draw mid-segment XY.



Compare $\angle AXY$ to $\angle ABC$ and compare $\angle AYX$ to $\angle ACB$. Without using a protractor, what would you guess the relationship between these two pairs of angles is?

What are the implications of this relationship?

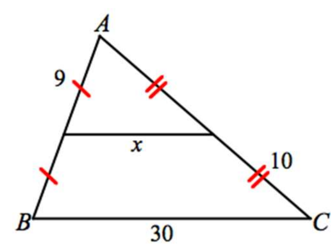
Properties of Mid-segments

- The mid-segment of a triangle is parallel to the third side of the triangle.
- The mid-segment of a triangle is half the length of the third side of the triangle.

Exercises

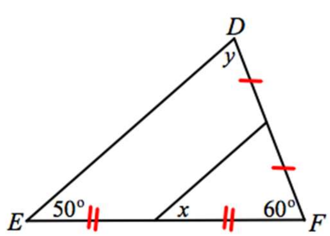
Apply what your know about the properties of mid-segments to solve the following:

1. *a.* Findx.

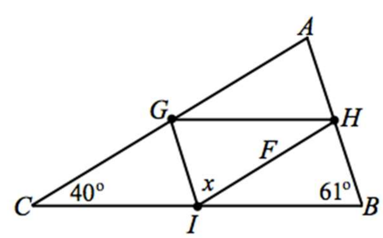


b. Find the perimeter of ΔABC

2. Find *x* and *y*.



3. Find *x*.



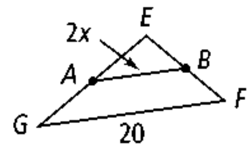
Problem

$\overline{AB}$ is a midsegment of ΔGEF. What is the value of x? $2AB = GF$

$2(2x) = 20$

$4x = 20$

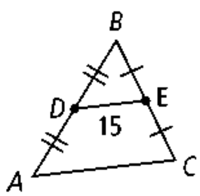
$x = 5$



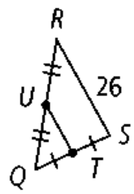
Exercises

Find the length of the indicated segment.

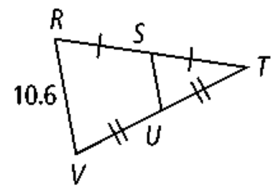
1. ***AC***



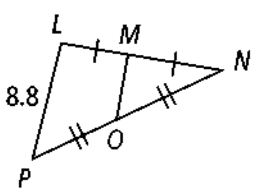
2. ***TU***



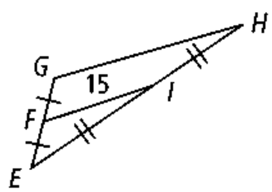
3. ***SU***



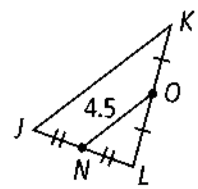
4. ***MO***



5. ***GH***

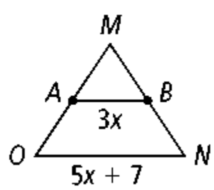


6. ***JK***

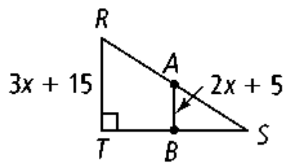


Algebra In each triangle, $\overline{AB}$ is a midsegment. Find the value of *x*.

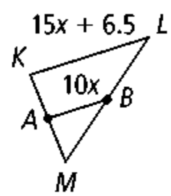
7.



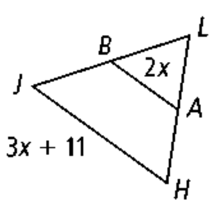
8.



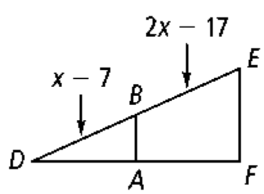
9.



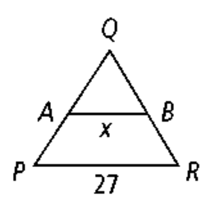
10.



11.

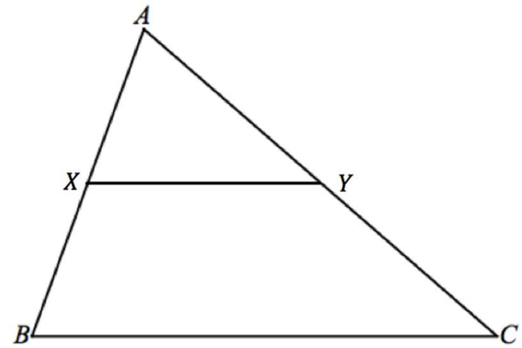


12.



Example 2

We are now going to prove the properties of mid-segments.



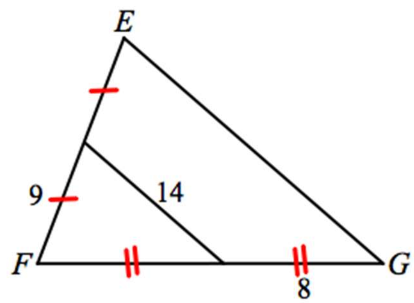
<u>Given</u>: XY is a mid-segment of $\triangle ABC$

<u>Prove</u>: $XY \parallel BC$ and $XY = \frac{1}{2}BC$

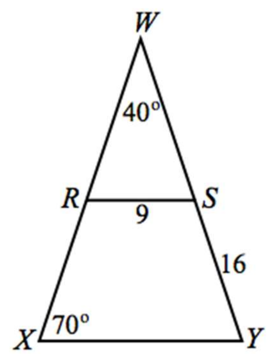
Statements	Reasons
1. XY is a mid-segment of $\triangle ABC$	1. Given
2. X is the midpoint of AB Y is the midpoint of AC	2. A mid-segment joins the midpoints
3. $AX \cong BX$ and $AY \cong CY$	3.
4.Extend XY to point G so that $YG = XY$ Draw GC	4.Auxiliary Lines
5. $\angle AYX \cong \angle CYG$	5.
6. $\triangle AYX \cong \triangle CYG$	6.
7. $\angle AXY \cong \angle CGY$, $AX \cong CG$	7.
8. $BX \cong CG$	8. Substitution
9. $AB \parallel CG$	9.
10. $BXGC$ is a parallelogram	10. One pair of opp. sides are □ and ≅
*11. $XY \parallel BC$	11. In a ⊔ , opposite sides are □
12. $XG \cong BC$	12. In a ⊔ , opposite sides are ≅
13. $XG = XY + YG$	13.
14. $XG = XY + XY$	14. Substitution
15. $BC = XY + XY$	15.
16. $BC = 2XY$	16. Substitution
*17. $XY = \frac{1}{2}BC$	17.

Homework

1. Find the perimeter of ΔEFG.

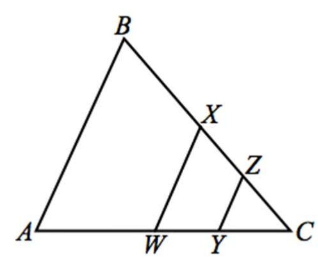


2. Find and label all of the missing sides and angles.



3. WX is a mid-segment of ΔABC, YZ is a mid-segment of ΔCWX and $BX = AW$.

a. What can you conclude about $\angle A$ and $\angle B$? Explain why.

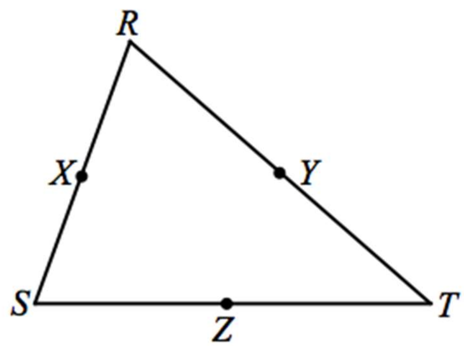


b. What is the relationship in length between YZ and AB?

Lesson 11: Points of Concurrency

Opening Exercise

The midpoints of each side of ΔRST have been marked by points *X*, *Y*, and *Z*.



a. Mark the halves of each side divided by the midpoint with a congruency mark. Remember to distinguish congruency marks for each side.

b. Draw mid-segments *XY*, *YZ*, and *XZ*. Mark each mid-segment with the appropriate congruency mark from the sides of the triangle.

c. What conclusion can you draw about the four triangles within ΔRST? Explain why.

d. State the appropriate correspondences between the four triangles within ΔRST.

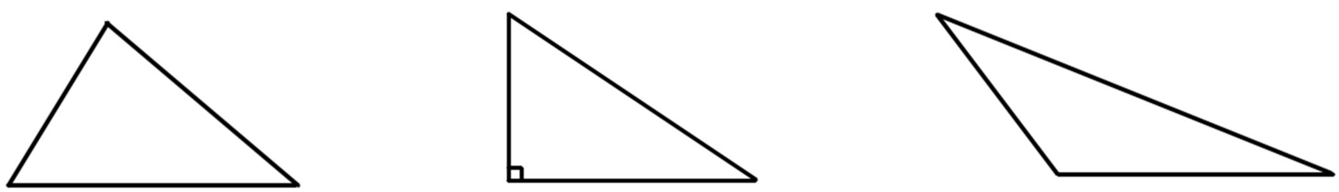
In Unit 1 we discussed two different points of concurrency (when 3 or more lines intersect in a single point).

Let's review what they are!

Circumcenter

- the point of concurrency of the 3 perpendicular bisectors of a triangle

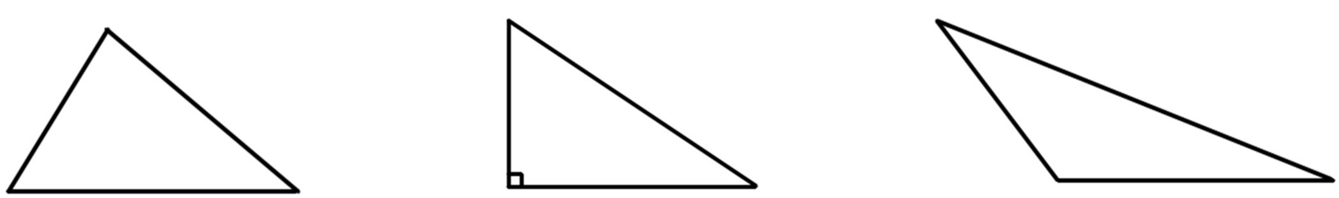
  Sketch the location of the circumcenter on the triangles pictured below:



Incenter

- the point of concurrency of the 3 angle bisectors of a triangle

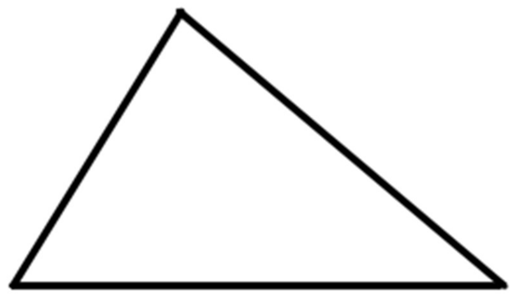
  Sketch the location of the incenter on the triangles pictured below:



Example 1

You will need a compass and a straightedge

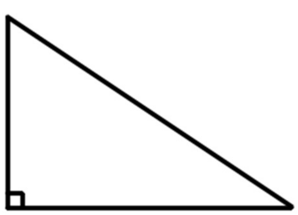
Construct the *medians* for each side of the triangle pictured below. A *median* is a segment connecting a vertex to the midpoint of the opposite side.

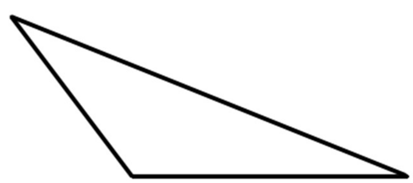


Vocabulary

- The point of intersection for 3 medians is called the ________________.
- This point is the *center of gravity* of the triangle.

We will use http://www.mathopenref.com/trianglecentroid.html to explore what happens when the triangle is right or obtuse. Sketch the location of the centroid on the triangles below:

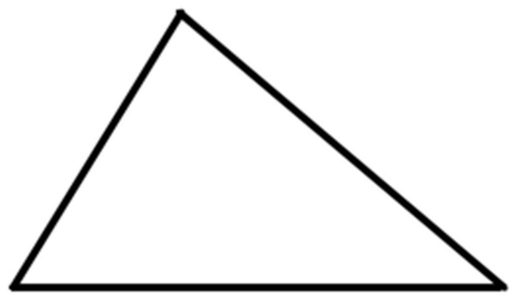




Example 2

You will need a compass and a straightedge

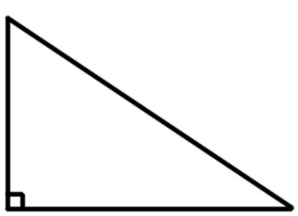
Construct the *altitudes* for each side of the triangle pictured below. An *altitude* is a segment connecting a vertex to the opposite side at a right angle. This can also be used to describe the height of the triangle.

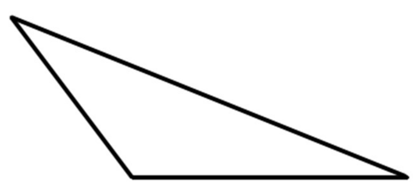


Vocabulary

- The point of intersection for 3 altitudes is called the _________________________________.

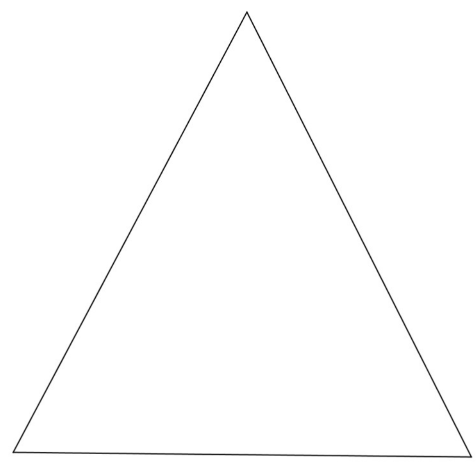
We will use http://www.mathopenref.com/triangleorthocenter.html to explore what happens when the triangle is right or obtuse. Sketch the location of the orthocenter on the triangles below:





Homework

Ty is building a model of a hang glider using the template below. To place his supports accurately, Ty needs to locate the center of gravity on his model.



a. Use your compass and straightedge to locate the center of gravity on Ty's model.

b. Explain what the center of gravity represents on Ty's model.

Lesson 12: Points of Concurrency II

Opening Exercise

Complete the table below to summarize what we did in Lesson 11. Circumcenter has been filled in for you.

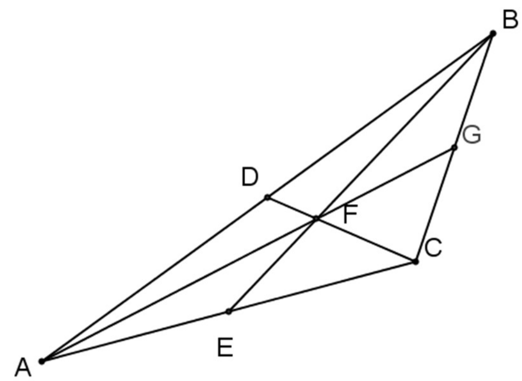
Point of Concurrency	**Types of Segments**	**What this type of line or segment does**	**Located Inside or Outside of the Triangle?**
Circumcenter	*Perpendicular Bisectors*	*Forms a right angle and cuts a side in half*	*Both; depends on the type of triangle*
Incenter			
Centroid			
Orthocenter			

Which two points of concurrency are located on the outside of an obtuse triangle?

What do these types have in common?

Example 1

A *centroid* splits the medians of a triangle into two smaller segments. These segments are always in a 2:1 ratio.



Label the lengths of segments *DF*, *GF* and *EF* as *x*, *y* and *z* respectively. Find the lengths of *CF*, *BF* and *AF*.

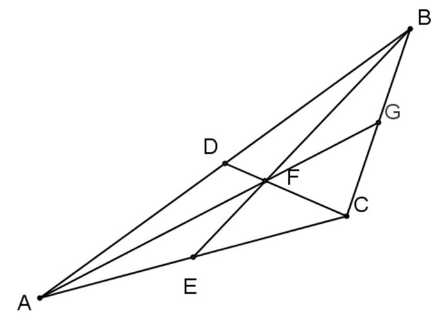
Exercises

1. In the figure pictured, $DF = 4$, $BF = 16$, and $GF = 10$. Find the lengths of:

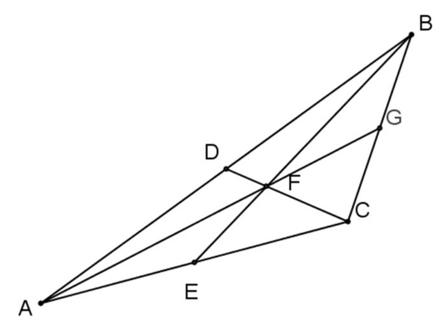
   a. CF

   b. EF

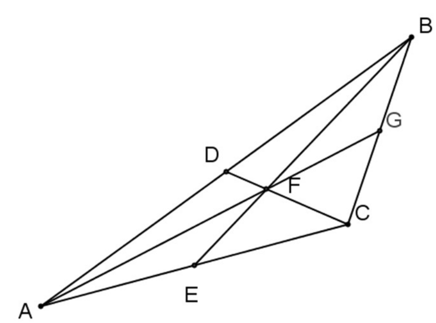
   c. AF



2. In the figure at the right, $EF = x + 3$ and $BF = 5x - 9$. Find the length of EF.



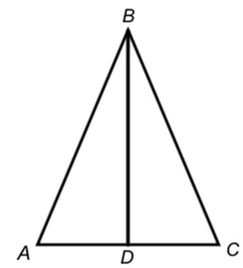
3. In the figure at the right, $DC = 15$. Find DF and CF.



We can now use medians and altitudes in triangle proofs!

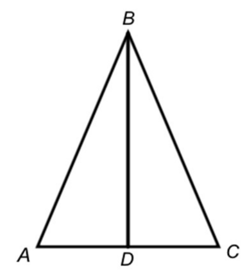
Here's how it looks:

Given: $\overline{BD}$ is the median of ΔABC



Statements	Reasons
1. $\overline{BD}$ is the median of ΔABC	1. Given
2.	2.
3.	3.

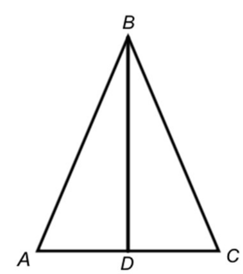
Given: $\overline{BD}$ is the altitude of ΔABC



Statements	Reasons
1. $\overline{BD}$ is the altitude of ΔABC	1. Given
2.	2.
3.	3.
4.	4.

Example 2

Given: $\overline{BD}$ is the median of ΔABC, $\overline{BD} \perp \overline{AC}$
Prove: $\angle A \cong \angle C$



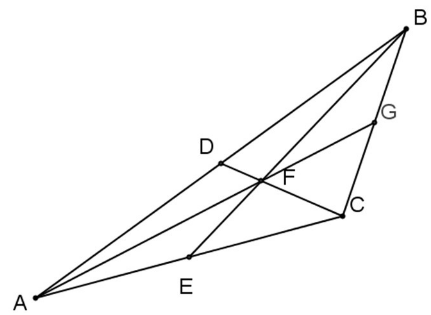
Homework

1. In the figure pictured, $DF = 3$, $BF = 14$, and $GF = 8$.
   Find the lengths of:

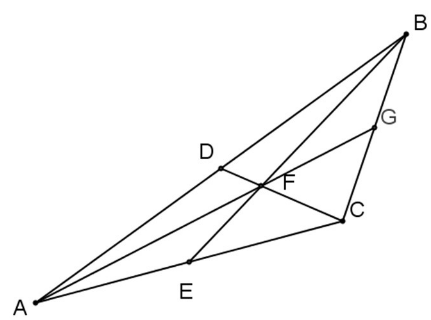
   a. CF

   b. EF

   c. AF



2. In the figure at the right, $GF = 2x - 1$ and $AF = 6x - 8$. Find the length of GA.



3. Given: $\overline{BD}$ is the altitude of ΔABC, $\angle ABD \cong \angle CBD$
   Prove: $\Delta ABD \cong \Delta CBD$