Unit 4
Similarity & Proofs

Lesson 1: Scale Drawings and Similarity

Opening Exercise

A common feature on smartphones is the ability to scale (enlarge or reduce) pictures:

Today we will review a concept that will give you the background on how the code for this feature was written, and the general steps the code must dictate.

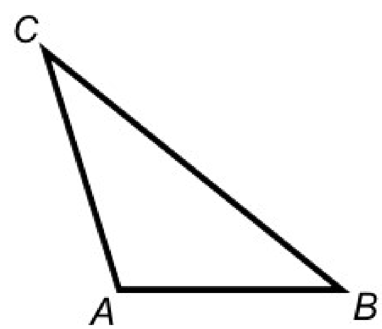
Using this picture of a bicycle:

Which of the images below appears to be a well-scaled image of the original? Why?

Example 1

You will need a compass and a straightedge

Create a scale drawing of $\triangle ABC$ with a scale factor of $r = 2$.



What is true about the sides of a well-scaled drawing?

What is true about the angles of a well-scaled drawing?

Vocabulary

- The constant of proportionality by which all lengths are scaled is called the

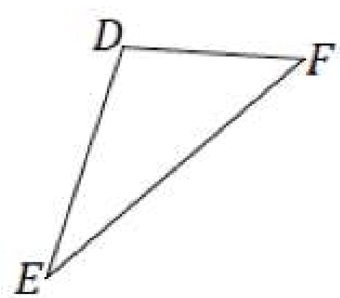
  ________________________________.

Exercise 1

You will need a compass and a straightedge

You are going to create a scale drawing of ΔDEF with a scale factor of $r = 3$. We are going to scale using different angles to see what the end result is.

Partner #1: Scale using angle E (extend sides ED and EF).
Partner #2: Scale using angle F (extend sides FD and FE).

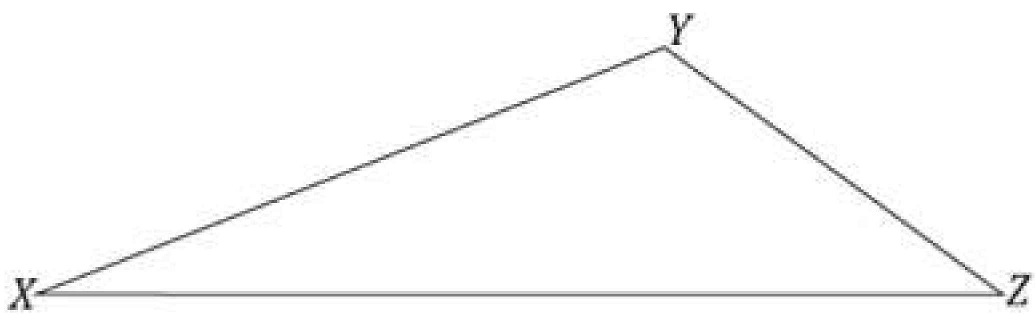


Compare your drawing with your partner's. Do the triangles look the same? Explain.

Example 2

You will need a compass and a straightedge

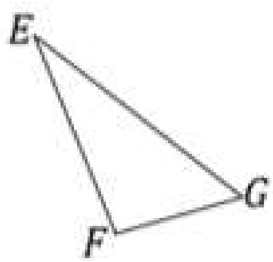
Create a scale drawing of ΔXYZ with a scale factor of $r = \frac{1}{2}$.



Exercise 2

You will need a compass and a straightedge

ΔEFG is provided below, and one angle of scale drawing $\Delta E'F'G'$ is also provided. Use construction tools to complete the scale drawing so that the scale factor is $r = 3$.

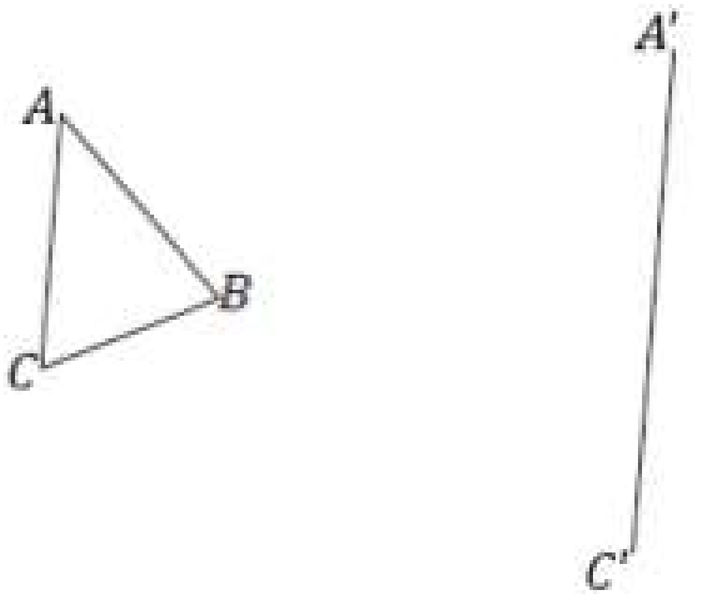


What properties do the scale drawing and the original figure share? Explain how you know.

Exercise 3

You will need a compass and a straightedge

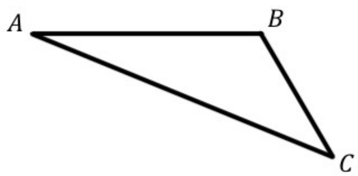
ΔABC is provided below, and one side of scale drawing $\Delta A'B'C'$ is also provided. Use construction tools to complete the scale drawing and determine the scale factor.



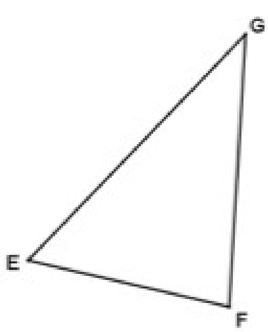
Homework

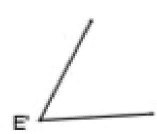
You will need a compass and a straightedge

1. Create a scale drawing of ΔABC with a scale factor of r = 4.



2 ΔEFG is provided below, and one angle of scale drawing $\Delta E'F'G'$ is also provided. Use construction tools to complete a scale drawing so that the scale factor is r = 2.





Lesson 2: Scale Drawings Using the Ratio Method

Opening Exercise

Based on what we did in Lesson 1, answer the following questions about scale drawings:

1. What is true about the sides of a well-scaled drawing?

2. What is true about the angles of a well-scaled drawing?

3. Does the location (or orientation) of the scale drawing impact the final image?

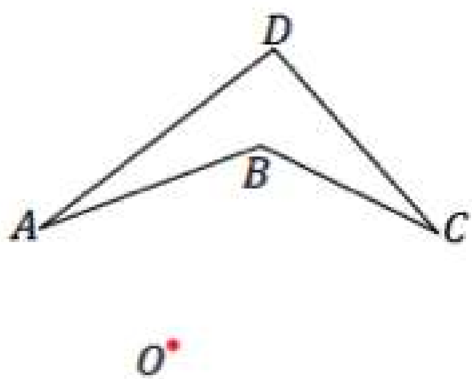
<u>Note</u>: When 2 figures have sides that are in proportion and angles that are congruent, they are called ______________________________ . We will define this more formally in a later lesson.

Example 1

You will need a compass and a straightedge

We are now going to create a scale drawing using the **Ratio Method.** This method of scaling is used when provided a scale factor AND a center point to scale from.

Create a scale drawing of the figure below using the Ratio Method about center O and scale factor of $r = 3$.



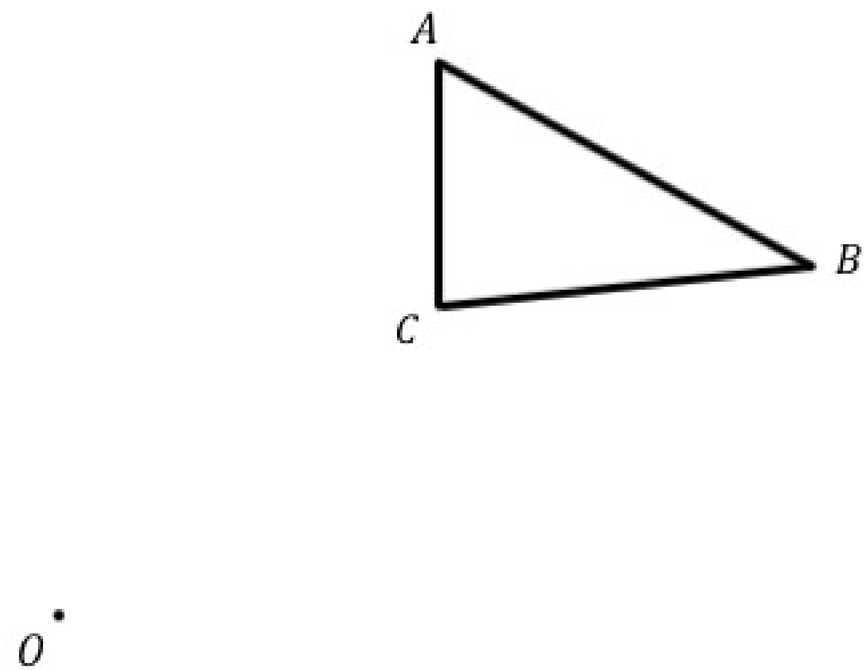
Vocabulary

	Define	Notation
Dilation		

Example 2

You will need a compass and a straightedge

Use the Ratio Method to create a scale drawing: $D_{O,\frac{1}{2}}$



What happens when the scale factor is less than 1 (and greater than 0)?

What happens when the scale factor is greater than 1?

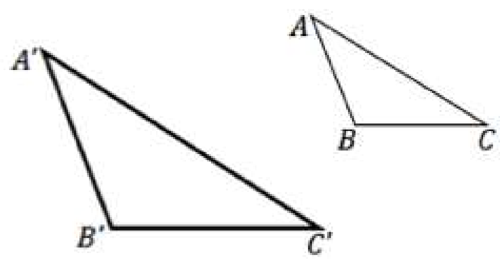
What would happen if the scale factor was equal to 1?

Note: *Scale factor is always a positive value because it is used when working with distance. A negative distance would not make sense!*

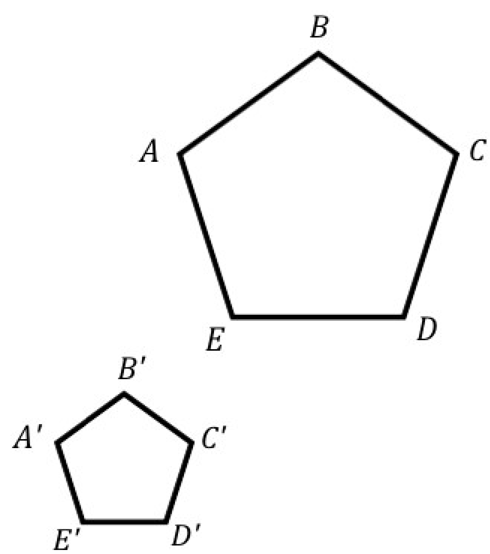
Example 3

Use your ruler to determine the location of center O used for the following scaled drawings. Then determine if the scale factor would be less than 1 or greater than 1.

a.



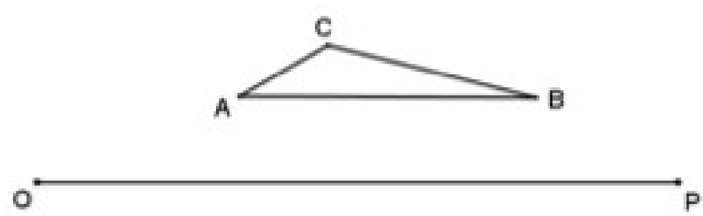
b.



Homework

You will need a compass and a straightedge

Use the Ratio Method to create two scale drawings: $D_{O,2}$ and $D_{P,2}$. Label the scale drawing with respect to center O as $\Delta A'B'C'$ and the scale drawing with respect to center P as $\Delta A''B''C''$.



What do you notice about the two scale drawings? Explain.

What rigid motion can be used to map $\Delta A'B'C'$ to $\Delta A''B''C''$?

Lesson 3: Dilations on the Coordinate Plane

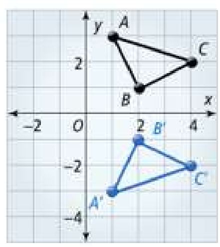
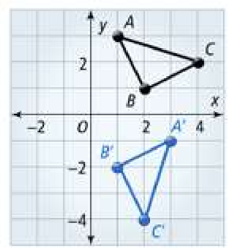
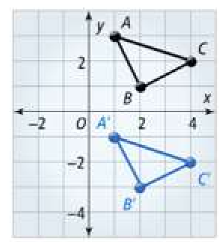
Opening Exercise

Recall from Unit 2:

A **transformation** is a change in the position, shape, or size of a figure.

A **rigid motion** is a transformation that changes only the position of the figure (length and angle measures are preserved).

Fill in the table below to identify the type of rigid motion being applied to create $\Delta A'B'C'$, the image of ΔABC.

			
Type of Rigid Motion:			

Is a **dilation** a type of transformation? Explain.

Is a **dilation** a rigid motion? Explain.

Example 1

As we saw in the first two lessons, when scaling or dilating, the figure gets larger or smaller. The scale factor determines this and keeps the figure in proportion.

If r = 3, what happens to the sides of the figure?

If r = ½, what happens to the sides of the figure?

If r = 1, what happens to the sides of the figure?

Based on these observations, what operation is used with dilations?

General Rule of Dilations: $D_{O,k}(x,y) =$

Exercises

1. What are the coordinates of the image of point (2, -4) under the dilation $D_{O,2}$?

2. What are the coordinates of the image of point (9, 15) under the dilation $D_{O,\frac{1}{3}}$?

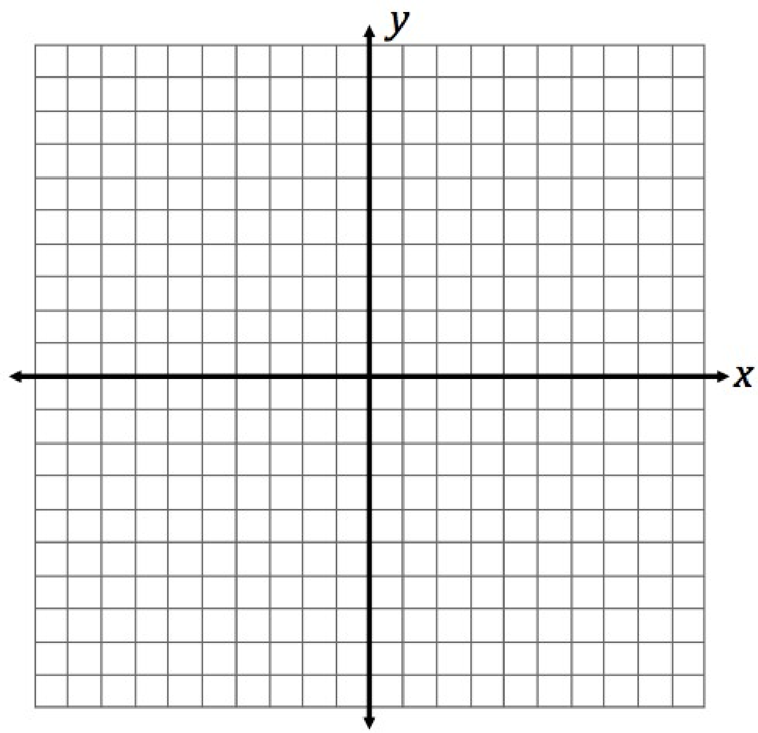
3. Find the scale factor for the dilation that maps (-4, 6) onto (-16, 24).

4. Find the scale factor for the dilation that maps (-4, 6) onto (-10, 15).

Example 2

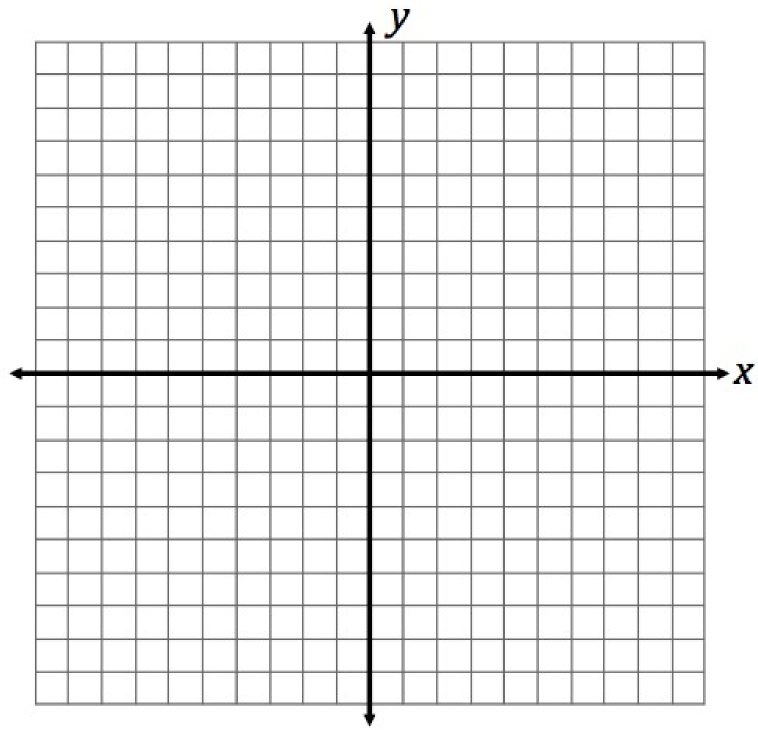
a. Graph ΔNAP with vertices $N(2, 8)$, $A(-6, 0)$ and $P(4, -2)$.

b. Graph and state the coordinates of $\Delta N'A'P'$, the image of ΔNAP after a dilation centered at the origin with a scale factor of ½.



Example 3

The coordinates of ΔABC are $A(1, 3)$, $B(-2, 2)$ and $C(0, -2)$. Graph and state the coordinates of $\Delta A''B''C''$, the result of the composite transformation $D_{O,2} \circ T_{3,-2}$



Homework

1. What are the coordinates of the image of point (-3, 2) under the dilation $D_{O,3}$?

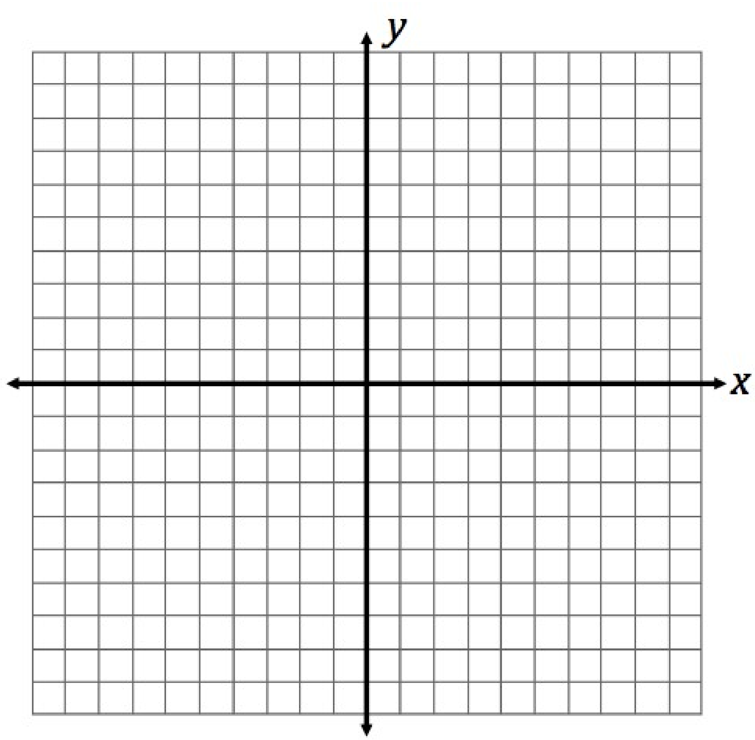
2. Find the scale factor for the dilation that maps (8, -2) onto (4, -1).

3. ΔBUG has vertices $B(2, 1)$, $U(-4, 4)$ and $G(-5, 2)$

   a. Graph and label ΔBUG on the axes provided.

   b. Graph and state the coordinates of $\Delta B'U'G'$, the image of ΔBUG after r_{x-axis}.

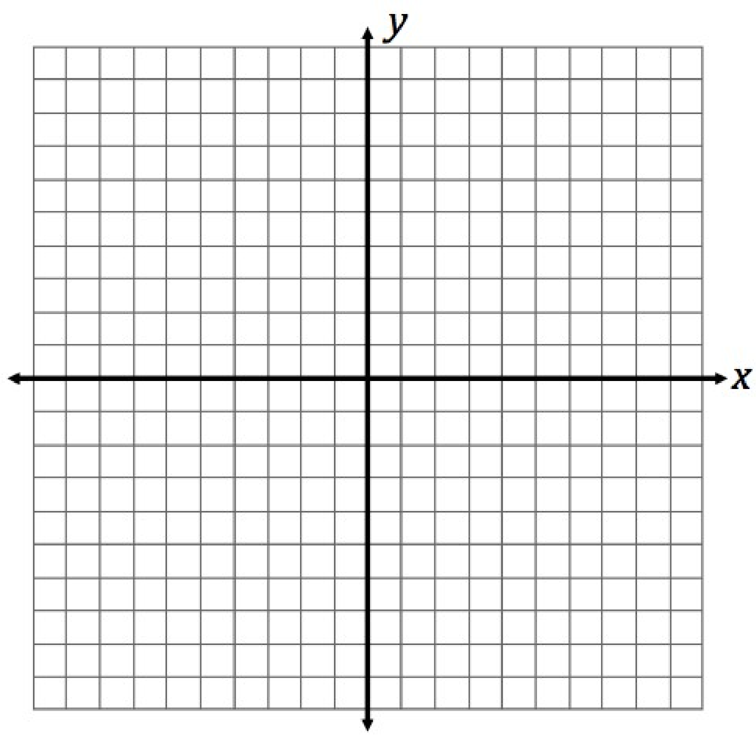
   c. Graph and state the coordinates of $\Delta B''U''G''$, the image of $\Delta B'U'G'$ after $D_{O,2}$.



Lesson 4: Dilations Mapping Segments, Lines, Rays and Circles

Opening Exercise

Segment $\overline{PQ}$ has endpoints $P(-3, 1)$ and $Q(3, 2)$. Graph segment $\overline{PQ}$ on the axes below and graph its image after a dilation from the origin with a scale factor of 3.



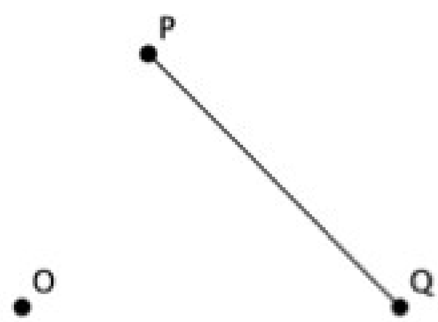
Is a dilated segment still a segment?

What is the relationship between $\overline{PQ}$ and its image after the dilation? (Two things!)

Example 1

You will need a compass and a straightedge

Dilate $\overline{PQ}$ by a scale factor of 2 from center O.



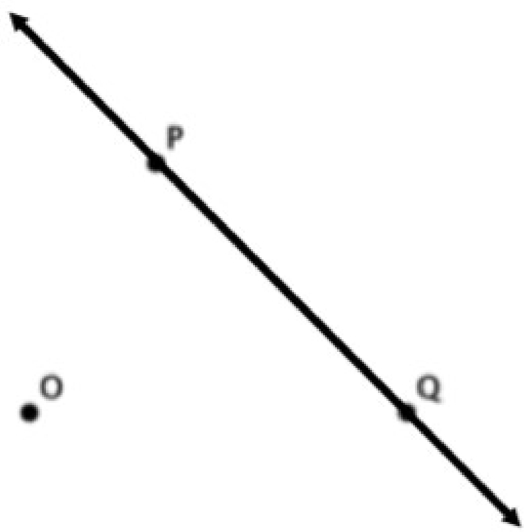
Is the image still a segment?

What is the relationship between $\overline{PQ}$ and its image after the dilation?

Example 2

You will need a compass and a straightedge

Dilate $\overleftrightarrow{PQ}$ by a scale factor of 2 from center O.



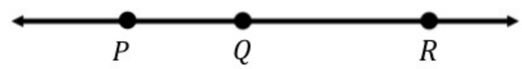
Is the image still a line?

What is the relationship between $\overleftrightarrow{PQ}$ and its image after the dilation?

Example 3

You will need a compass

Dilate $\overline{PQ}$ by a scale factor of 3 from center *R*.



How much longer is $\overline{P'Q'}$ than $\overline{PQ}$?

What do you notice about line PQ vs. line $P'Q'$?

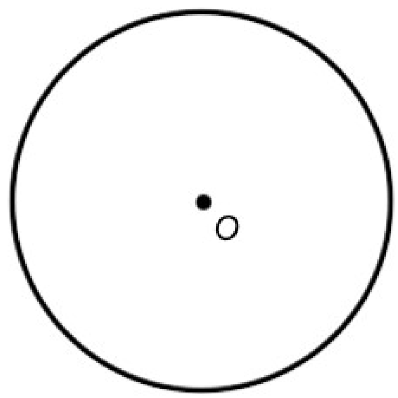
Summary

- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- A dilation takes a line not passing through the center of dilation to a parallel line.
- A dilation of a line passing through the center of dilation leaves the line unchanged.

Example 4

You will need a compass and a straightedge

Dilate circle O by a scale factor of 2 with O being the center of dilation.



What do you notice about the centers of the circle given and its image after the dilation?

What is true about the radii?

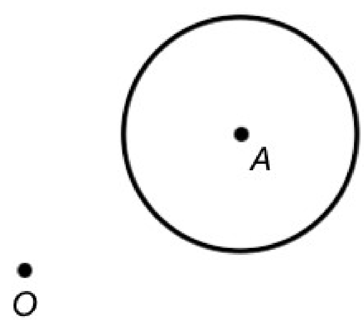
Vocabulary

Define	Diagram
Concentric Circles	

Example 5

You will need a compass and a straightedge

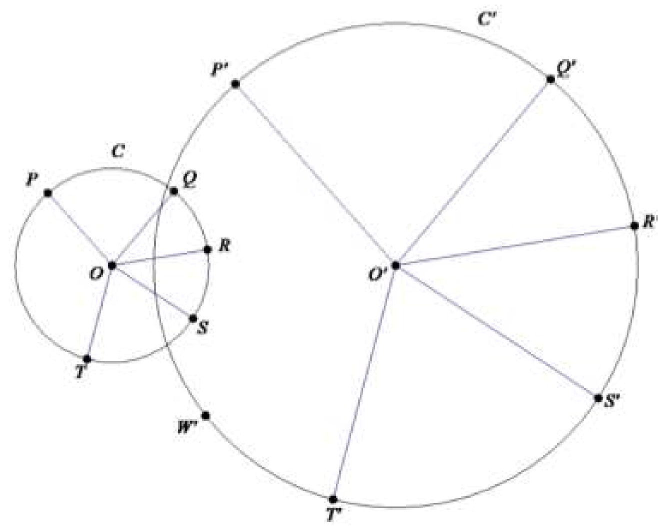
Dilate circle A by a scale factor of 2 with O being the center of dilation.



What is the relationship between circle A's radius and the radius of its image after the dilation?

Example 6

Find the center of dilation that would map circle O to circle O'.

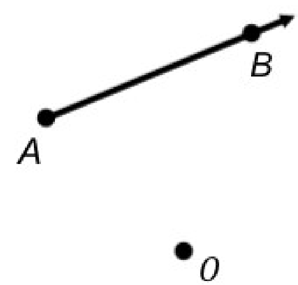


Use your center of dilation to locate point W on circle O.

Homework

You will need a compass and a straightedge

Dilate $\overrightarrow{AB}$ by a scale factor of 4 from center O.

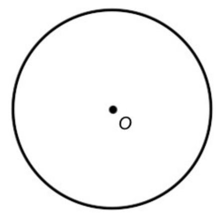


Is the image still a ray?

What is the relationship between ray *AB* and its image after the dilation?

What is the relationship between segment *AB* and segment *A'B'*?

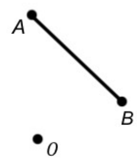
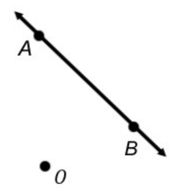
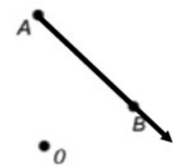
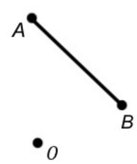
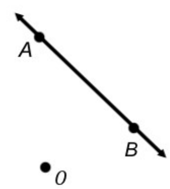
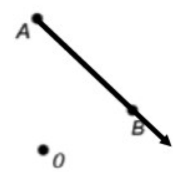
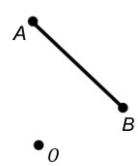
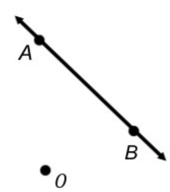
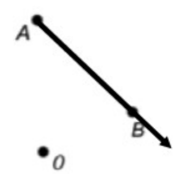
2. Dilate circle O by a scale factor of 3 with O being the center of dilation. What kind of circles are formed?



Lesson 5: Dilations Mapping Angles

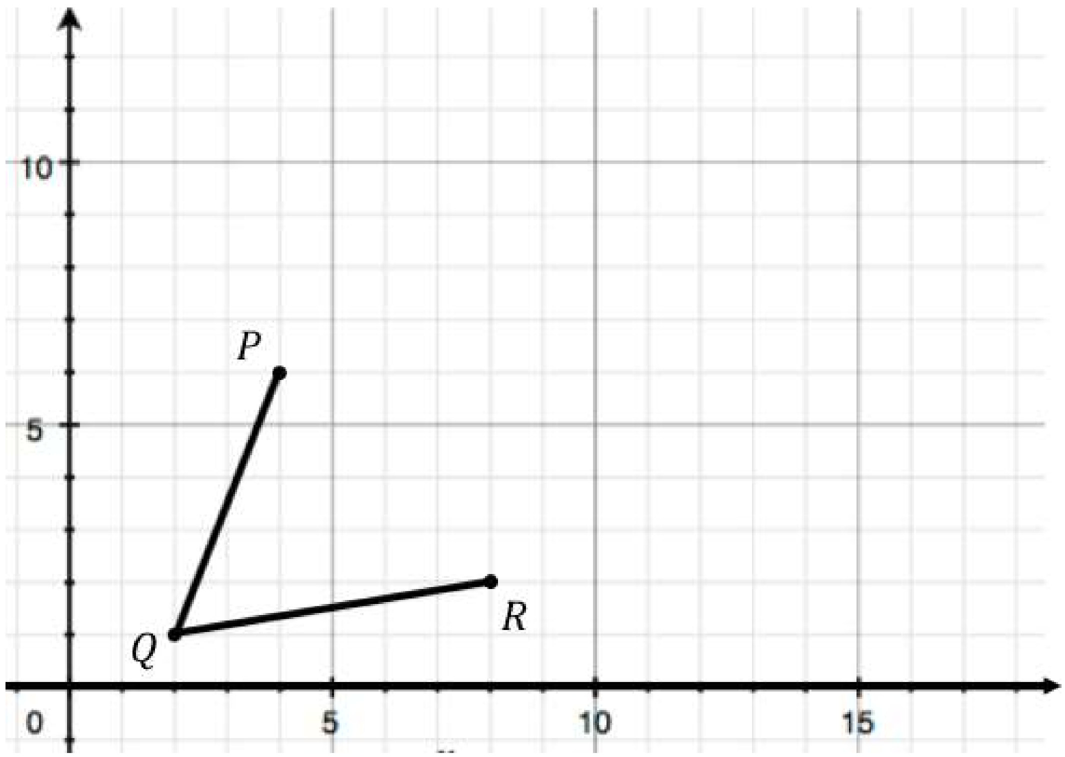
Opening Exercise

Sketch the image of the following after dilating from center O with the given scale factor:

	Segment	**Line**	**Ray**
$r > 1$			
$r < 1$			
$r = 1$			

Example 1

Given $\angle PQR$ as pictured on the graph below, draw $\angle P'Q'R'$, the image of $\angle PQR$ after a dilation $D_{O,2}$.



What do we know about the segment relationships?

Using this knowledge about segment relationships, what can we conclude about the angles?

Example 2

The line $y = 2x - 4$ is dilated by a scale factor of 3/2 and centered at the origin. Write the equation that represents the image of the line after the dilation.

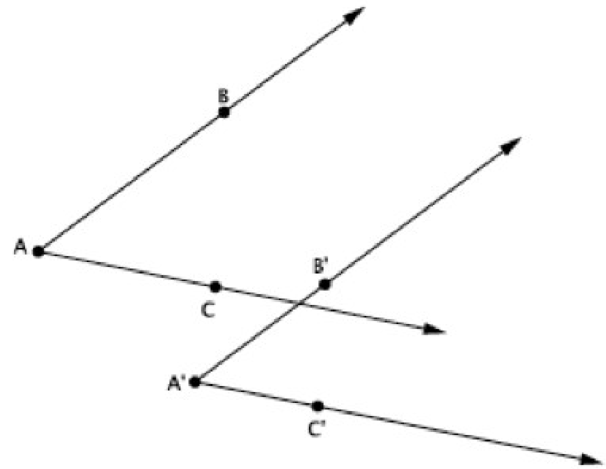
Example 3

The equation of line h is $2x + y = 1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?

Example 4

<u>Given</u>: A dilation from center O by scale factor r maps $\angle BAC$ to $\angle B'A'C'$.

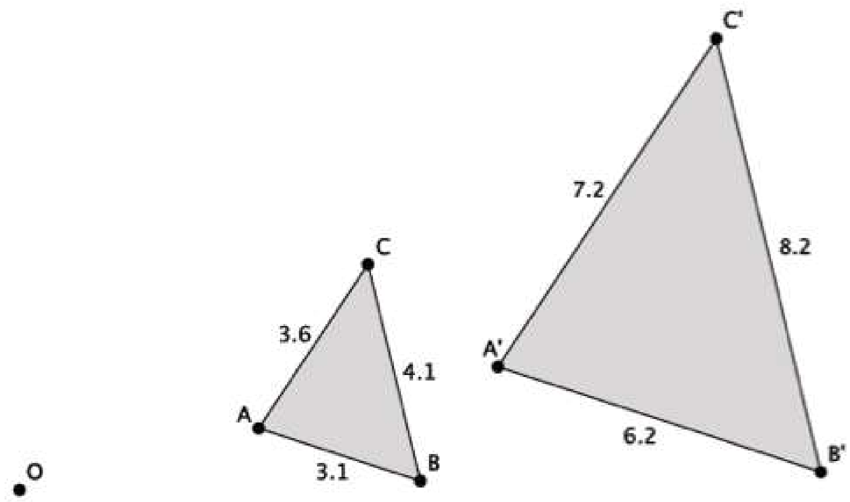
<u>Prove</u>: $\angle A \cong \angle A'$



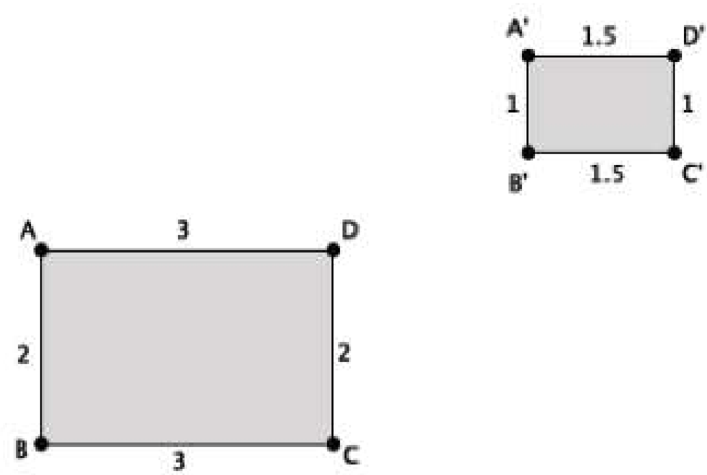
Example 5

Find the scale factor for each of the diagrams below and show that the scale factor holds true for each side of the figure.

a.



b.

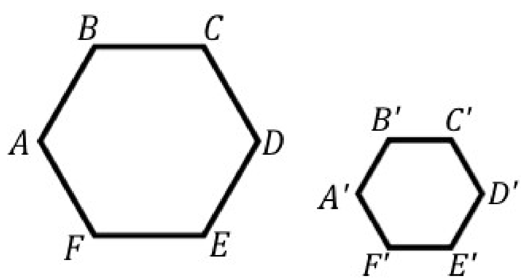


Homework

1. Point A has coordinates (4, -6). Find A', the image of A, after $D_{O,\frac{3}{2}}$.

2. Write the rule for the dilation that maps (-3, -2) onto (-15, -10).

3. Find O, the center of dilation for the diagram pictured below. Would the scale factor be less than 1 or greater than 1?



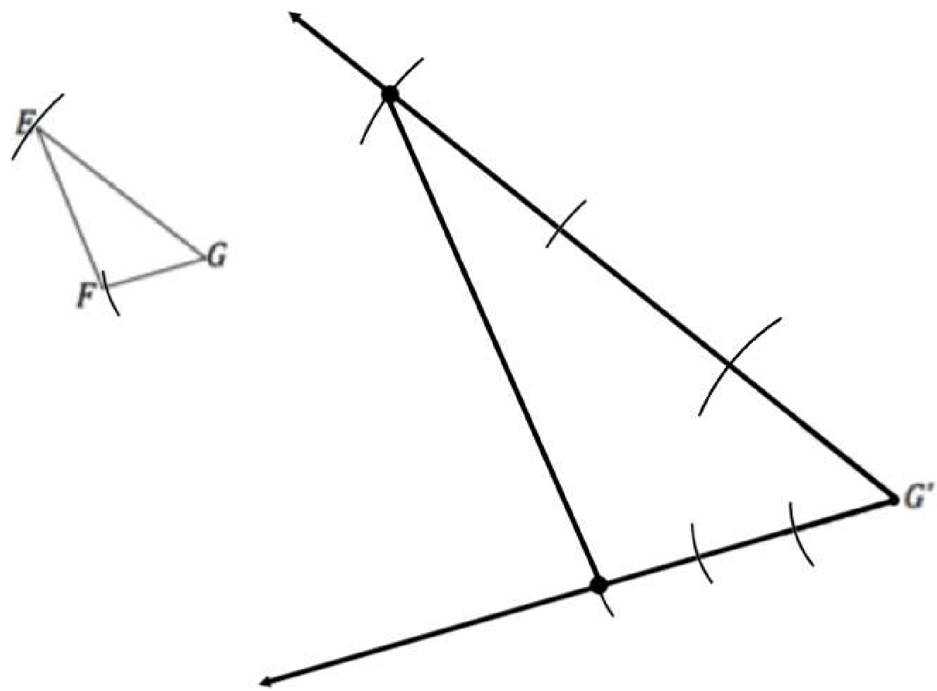
4. Create the image of $\overline{AB}$ after $D_{O,2}$.

Lesson 6: Similarity Transformations

Opening Exercise

Shown below is from page 5 of the notes when we were doing scale drawings. The work has already been done for you!!

ΔEFG is provided below, and one angle of scale drawing $\Delta E'F'G'$ is also provided. Use construction tools to complete the scale drawing so that the scale factor is r = 3.



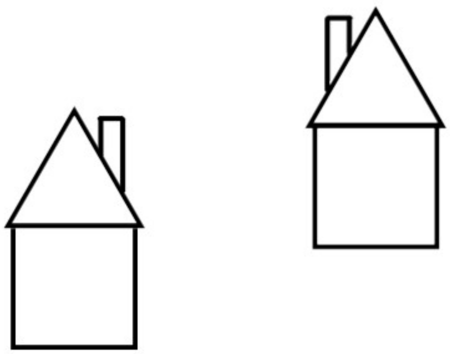
What is the relationship between the sides of the original figure and the scaled drawing?

What is the relationship between the angles of the original figure and the scaled drawing?

*<u>Recall</u>: These figures can be described as **similar.***

Example 1

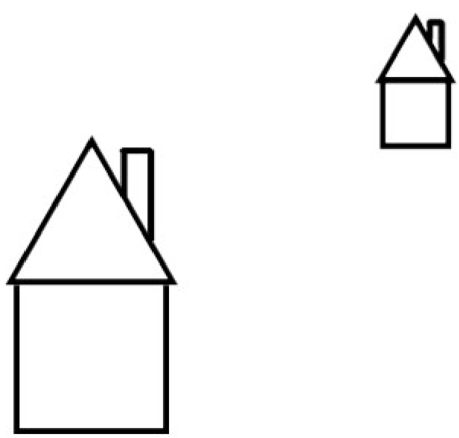
Define the transformations that would map the pre-image on the left to the image on the right.



What do we know about the figures?

Example 2

Define the transformations that would map the pre-image on the left to the image on the right



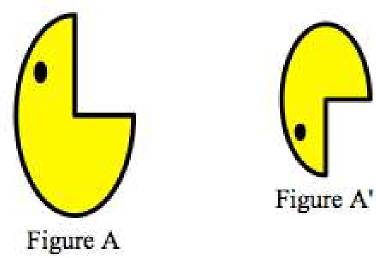
What do we know about the figures?

Vocabulary

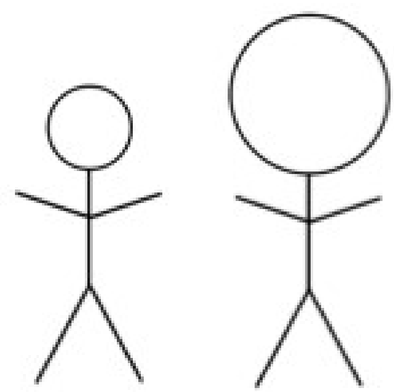
Define	Diagram
Similarity Transformation	
Similar	

Exercises

1. Figure A' is similar to Figure A. Which transformations compose the similarity transformation that maps Figure A onto Figure A'?

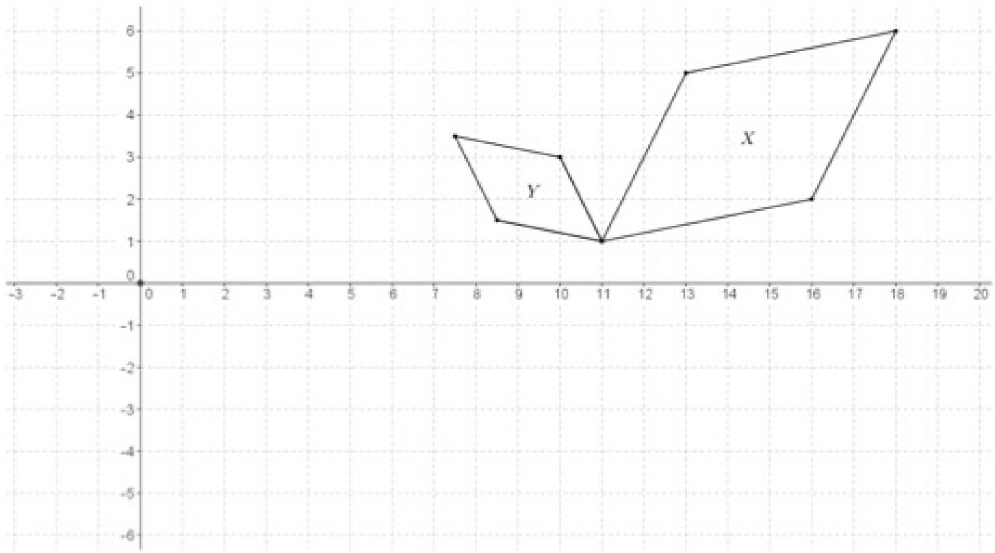


2. Is there a sequence of dilations and basic rigid motions that takes the small figure to the large figure? Explain.



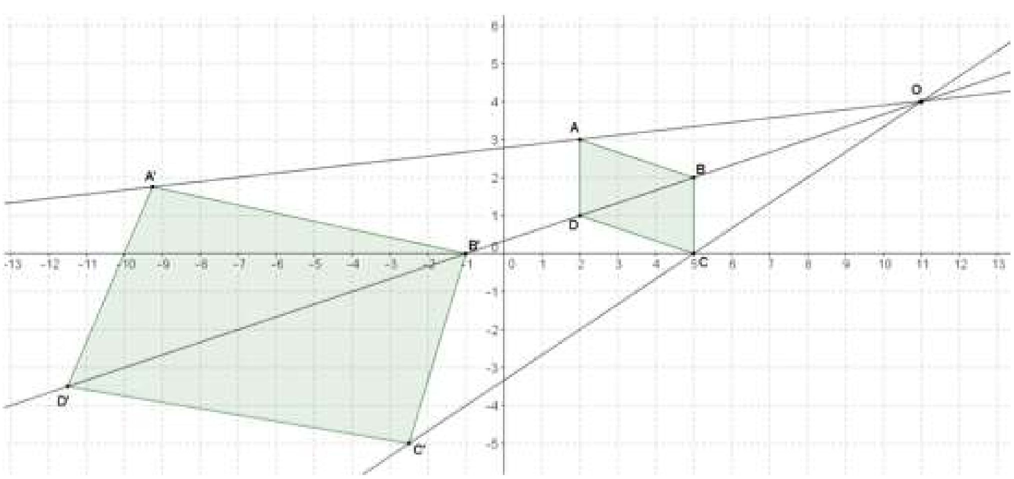
Example 3

Given the coordinate plane shown, identify a similarity transformation, if one exists, mapping X onto Y. If one does not exist, explain why.



Example 4

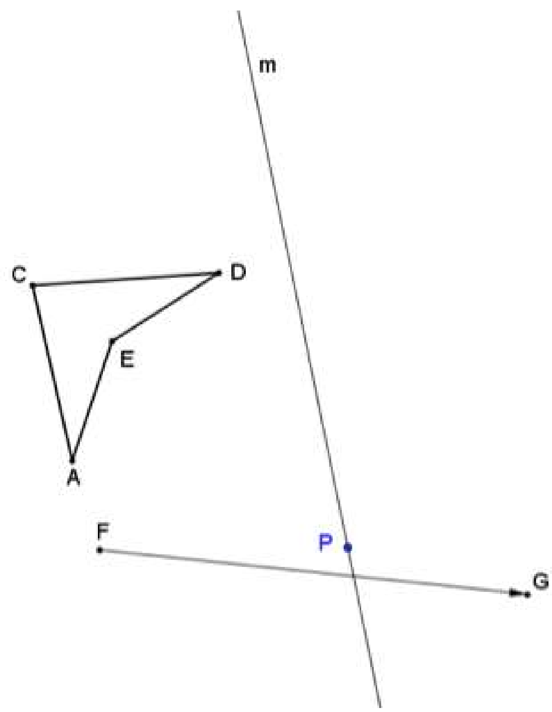
Is the diagram pictured below an example of a dilation? Explain.



Example 5

You will need a compass and a straightedge

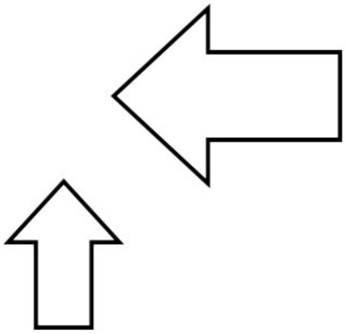
Construct *A''C''D''E''*, the image of *ACDE* after $D_{P,2} \circ r_m(ACDE)$.



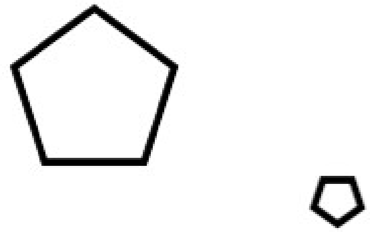
Homework

1. Identify the similarity transformations that maps the figure on the left to the figure on the right.

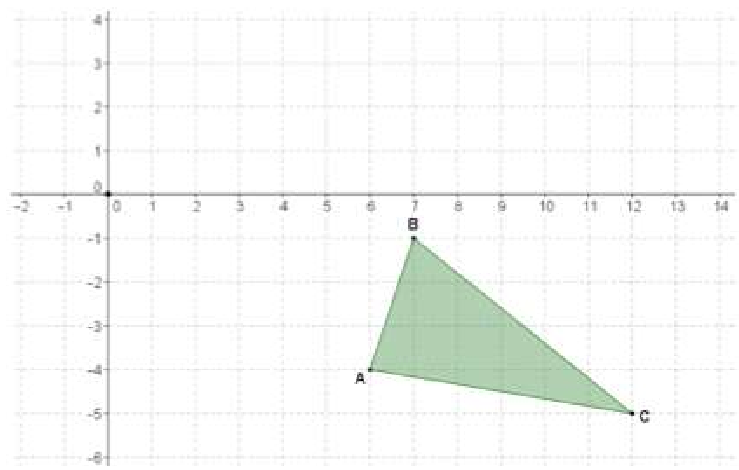
   a.



   b.



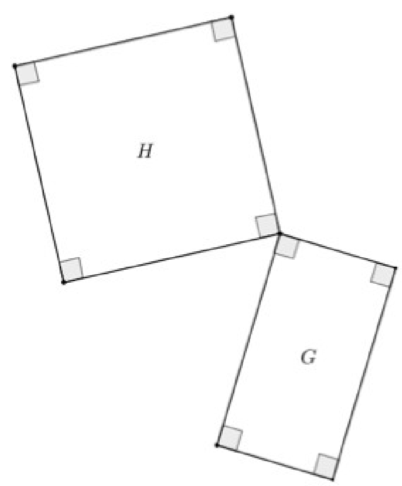
2. Given ΔABC as pictured.



   a. Graph and state the coordinate of $\Delta A'B'C'$, the image of ΔABC after translating vertex A to the origin.

   b. Graph and state the coordinates of $\Delta A''B''C''$, the image of $\Delta A'B'C'$ after $D_{O,1/3}$

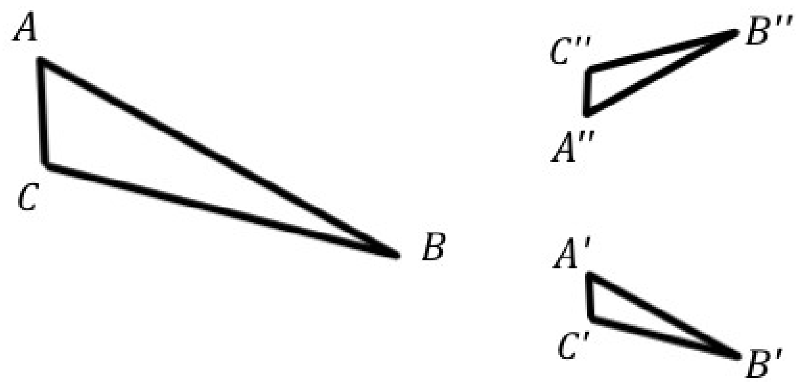
3. Given the diagram pictured to the right, identify a similarity transformation, if one exists, that maps G onto H. If one does not exist, explain why.



Lesson 7: Similarity Criteria for Triangles - AA

Opening Exercise

Write the similarity transformation shown below:



	$\Delta ABC \sim \Delta ABC$	$\Delta ABC \sim \Delta A'B'C'$	$\Delta A'B'C' \sim \Delta A''B''C''$
Identify the Scale Factor of the Dilation			

Using the chart above answer the following:

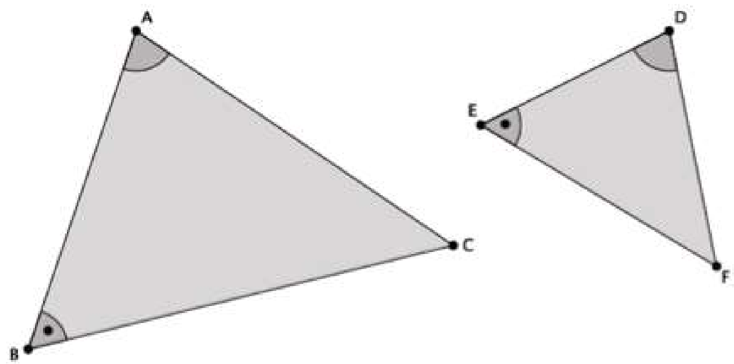
a. $\Delta ABC \sim \Delta ABC$ is an example of what property?

b. If $\Delta ABC \sim \Delta A'B'C'$, is $\Delta A'B'C' \sim \Delta ABC$? Identify the property.

c. If $\Delta ABC \sim \Delta A'B'C'$ and $\Delta A'B'C' \sim \Delta A''B''C''$, is $\Delta ABC \sim \Delta A''B''C''$? Identify the property.

Example 1

Given ΔABC □ ΔDEF, we are going to show why only two pairs of congruent angles are needed to prove triangle similarity.



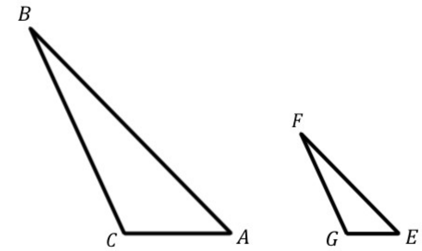
What is the similarity transformation that would map ΔABC onto ΔDEF?

What scale factor would we use for the dilation? Show a sketch of the dilation.

What is the relationship between the dilated triangle, $\Delta AB'C'$, and ΔDEF? How do you know?

Based on what was given, what is the **only** information we need to prove triangles are similar?

Criteria for Proving Similarity:

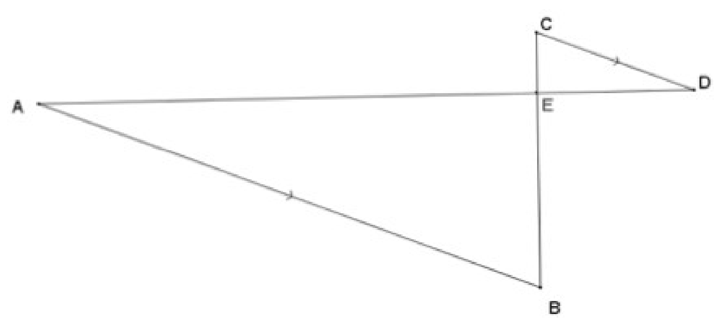


Triangle Similarity Criteria	Angle and/or Segment Relationships	Describe
AA		Two triangles are similar if two angles of one triangle are congruent to two corresponding angles of the other triangle.
SAS **(for similarity)**		Two triangles are similar if two pairs of corresponding sides are in proportion and the angles between them are congruent.
SSS **(for similarity)**		Two triangles are similar if the three sets of corresponding sides are in proportion.

Example 2

<u>Given</u>: $\overline{AD}$ and $\overline{BC}$ intersect at E, and $\overline{AB} \parallel \overline{CD}$

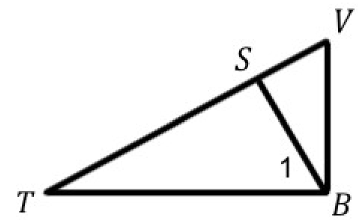
a. <u>Prove</u>: $\Delta ABE \sim \Delta DCE$



b. What similarity transformation maps ΔABC onto ΔDEF?

Example 3

<u>Given</u>: $\overline{BS} \perp \overline{TV}$ and $\angle 1 \cong \angle V$

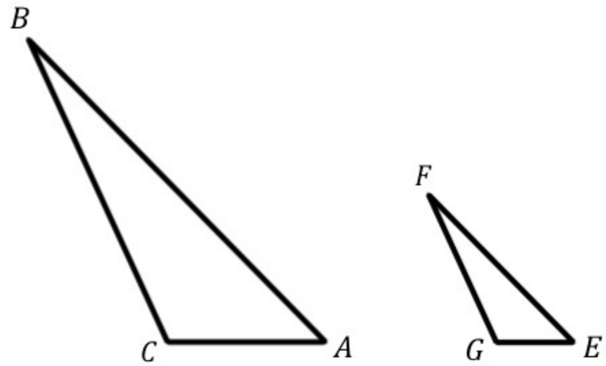


a. <u>Prove</u>: $\Delta TSB \sim \Delta BSV$

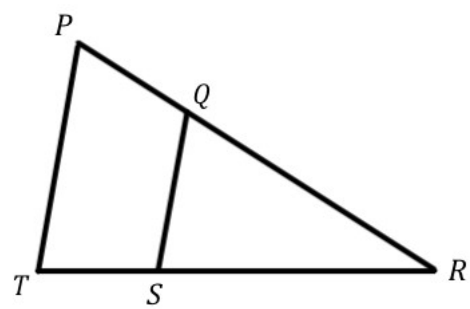
b. What similarity transformation maps ΔBSV onto ΔTSB?

Homework

1. In the diagram below $\Delta ABC \sim \Delta EFG$, $m\angle C = 4x + 30$ and $m\angle G = 5x + 10$. Determine the value of x.



2. Given: $\overline{PT} \parallel \overline{QS}$



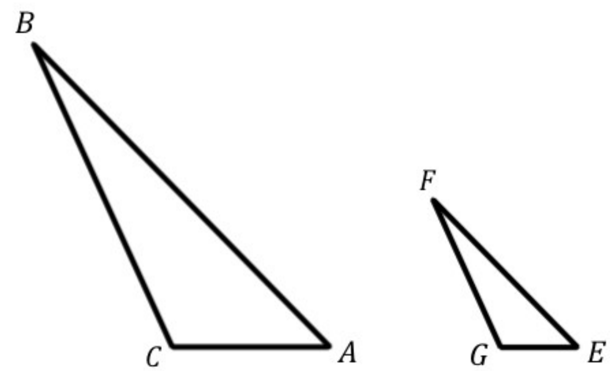
   a. Prove: $\Delta PRT \sim \Delta QRS$

   b. What similarity transformation maps ΔPRT onto ΔQRS?

Lesson 8: Similarity Criteria for Triangles – SAS and SSS

Opening Exercise

In the diagram below, $\Delta ABC \sim \Delta EFG$, $\angle C = 120°$, $\angle F = 15°$, $\overline{AB} = 20$, $\overline{EF} = 10$ and $\overline{AC} = 8$.

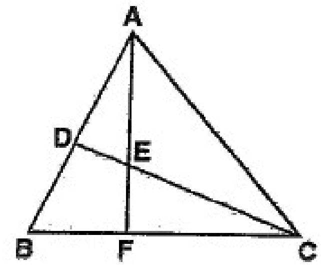


a. Find $m\angle E$.

b. Find the length of $\overline{EG}$.

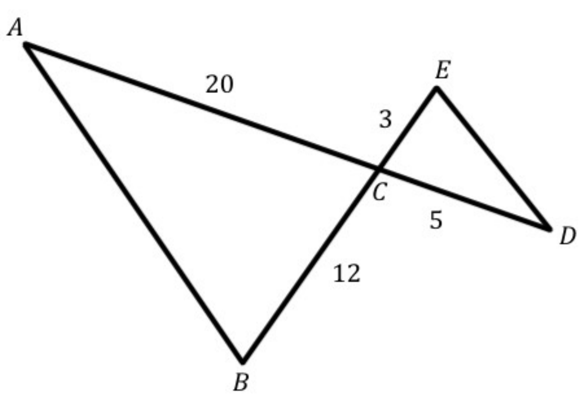
Example 1

<u>Given</u>: $\overline{CD}$ and $\overline{AF}$ are altitudes of ΔABC
<u>Prove</u>: $\overline{CE} \cdot \overline{ED} = \overline{AE} \cdot \overline{EF}$



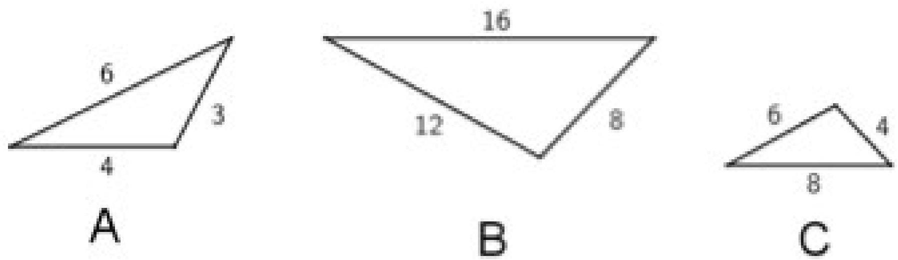
Example 2

Given the diagram below, is $\Delta ABC \sim \Delta DEC$? Explain your answer.



Example 3

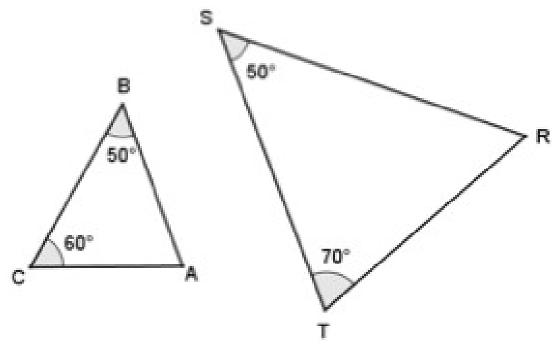
Which of the three triangles, if any, are similar and why?



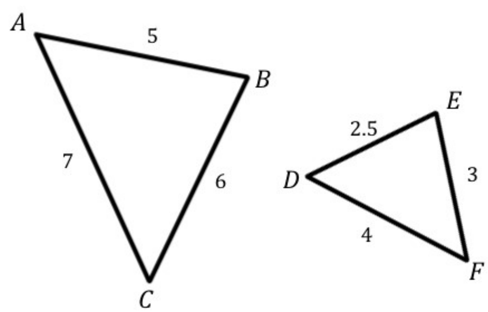
Homework

For each given pair of triangles, determine if the triangles are similar and provide your reasoning. If the triangles are similar, write a similarity statement relating the triangles.

a.



b.



c.