Unit 5
Applying Similarity of Triangles

Lesson 1: Proof of the Triangle Side Splitter Theorem

Opening Exercise

We are going to construct a proof designed to demonstrate the following theorem:

A line segment parallel to one side of a triangle divides the other two sides proportionally.

a. Fill in the hypothesis and conclusion:

If ____ ,

then ____ .

b. Draw a diagram:

c. Prove the theorem:

d. What similarity transformation maps the smaller triangle onto the larger triangle?

Example 1

We are now going to prove the converse of the theorem from the Opening Exercise:

A line segment that divides two sides of a triangle proportionally is parallel to the third side.

a. Fill in the hypothesis and conclusion:

If ____ ,

then ____ .

b. Draw a diagram:

c. Prove the converse:

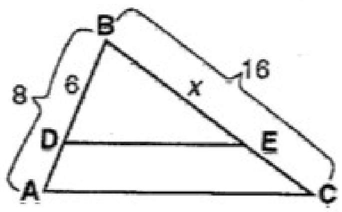
We have now proved the **Triangle Side Splitter Theorem**, which states:

A line segment splits two sides of a triangle proportionally if and only if it is parallel to the third side.

Exercises

In 1-3, find x given that $\overline{DE} \parallel \overline{AC}$.

1



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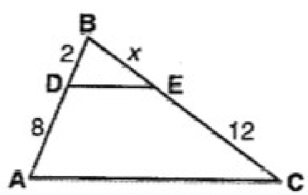

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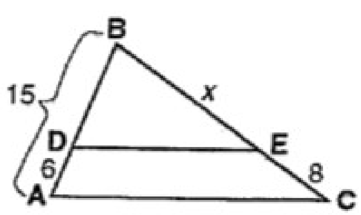


Lesson 2: Applying the Triangle Side Splitter Theorem

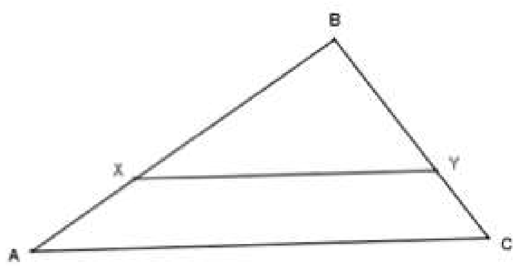
Opening Exercise

Yesterday we proved the Triangle Side Splitter Theorem, which states:

> *A line segment splits two sides of a triangle proportionally if and only if it is parallel to the third side.*

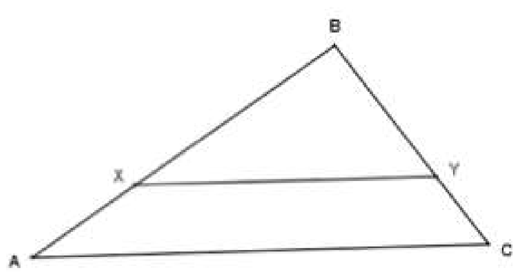
Using this Theorem, answer the following:

1. If $\overline{XY} \parallel \overline{AC}$, $\overline{BX} = 4$, $\overline{BA} = 5$, and $\overline{BY} = 6$, what is $\overline{BC}$?



Not drawn to Scale

2. If $\overline{XY} \parallel \overline{AC}$, $\overline{BX} = 9$, $\overline{BA} = 15$, and $\overline{BY} = 15$, what is $\overline{YC}$?



Not drawn to Scale

The Triangle Side Splitter Theorem allows us to solve using proportions, namely:

$$\frac{\text{upper left side}}{\text{lower left side}} = \frac{\text{upper right side}}{\text{lower right side}}$$

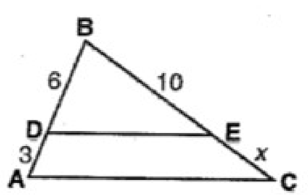
Since the upper side and lower side are in proportion, we can extend this to the whole segments as well:

$$\frac{\text{upper left side}}{\text{whole left side}} = \frac{\text{upper right side}}{\text{whole right side}} \qquad \frac{\text{lower left side}}{\text{whole left side}} = \frac{\text{lower right side}}{\text{whole right side}}$$

Exercises

In 1-2, find x given that $\overline{DE} \parallel \overline{AC}$.

1.



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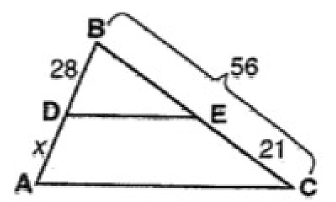
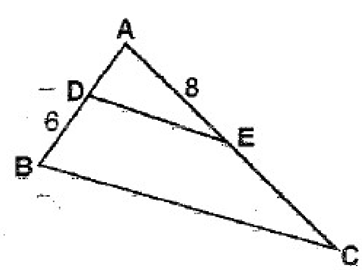
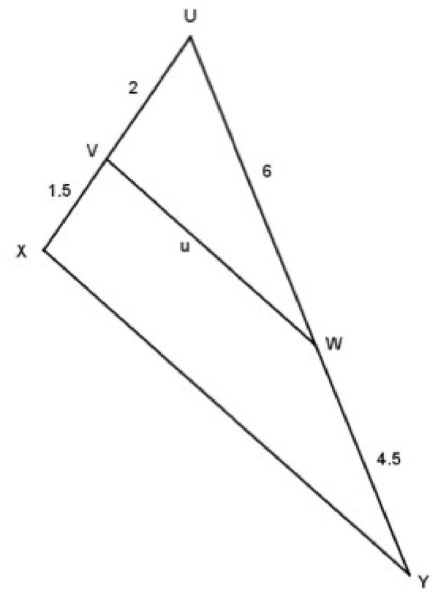


3. In the accompanying diagram, $\overline{DE} \parallel \overline{BC}$, $DB = 6$ and $AE = 8$. If EC is *three times AD*, find AD.



Example 1

Given the diagram pictured to the right, we can come up with the following conclusions. Explain each conclusion.



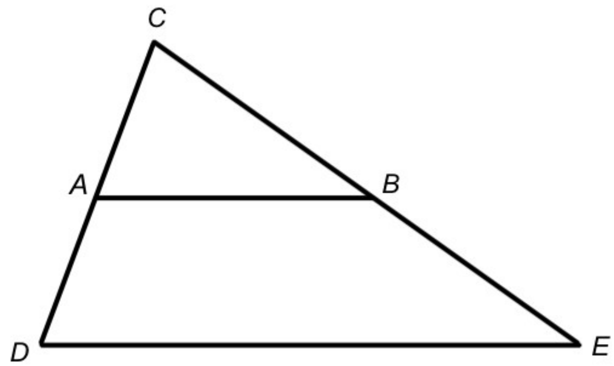
a. $\overline{VW} \parallel \overline{XY}$

b. $\angle Y \cong \angle UWV$ and $\angle X \cong \angle UVW$

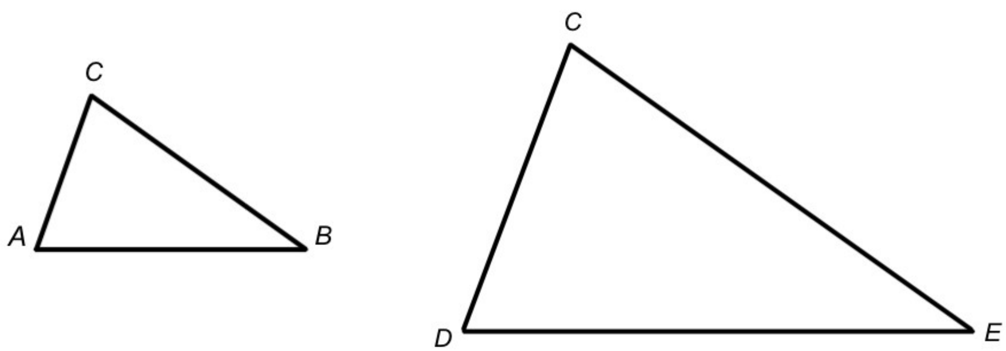
c. ΔUXY is a scale drawing of ΔUVW with a scale factor of $\frac{7}{4}$

d. $\Delta UXY \sim \Delta UVW$ and therefore $\overline{XY} = \frac{7}{4}u$

The Triangle Side Splitter Theorem allows us to work with the bases of the triangles as well.



Since we know $\Delta ACB \sim \Delta DCE$, its best to look at these triangles separately:

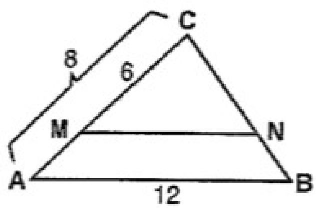


$$\frac{AC}{DC} = \frac{AB}{DE} \qquad \text{or} \qquad \frac{\textit{upper left side}}{\textit{whole left side}} = \frac{\textit{upper base}}{\textit{whole base}}$$

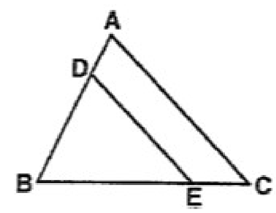
We cannot use the lower sides when working with bases!!!

Exercises

1. In the diagram below of ΔABC, $\overline{MN} \parallel \overline{AB}$, $AC = 8$, $AB = 12$, and $CM = 6$. Find the length of MN.

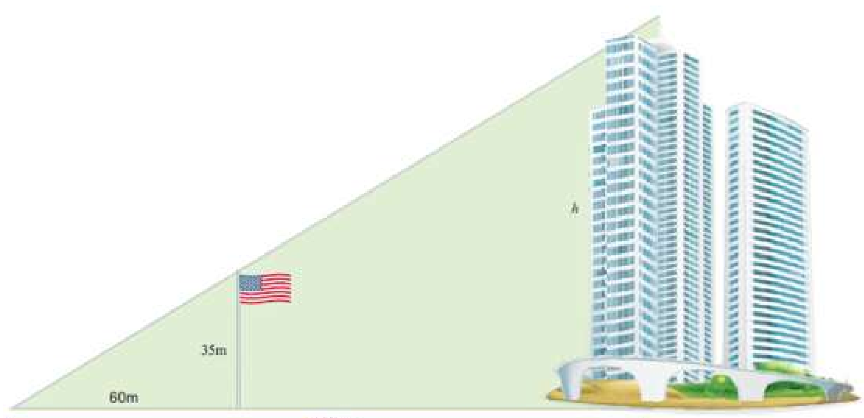


2. In the diagram below of ΔABC, $\overline{DE} \parallel \overline{AC}$, $DB = 6$, $AD = 2$, and $DE = 9$. Find AC.



3. A vertical pole, 15 feet high, casts a shadow 12 feet long. At the same time, a nearby tree casts a shadow 40 feet long. What is the height of the tree?

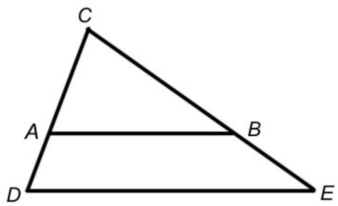
4. In the diagram pictured, a large flagpole stands outside of an office building. Josh realizes that when he looks up from the ground, 60 m away from the flagpole, that the top of the flagpole and the top of the building line up. If the flagpole is 35 m tall, and Josh is 170 m from the building, how tall is the building to the *nearest tenth*?



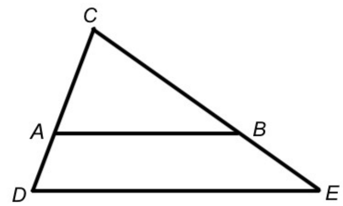
Homework

In 1-4, $\overline{AB} \parallel \overline{DE}$ and triangles have not been drawn to scale.

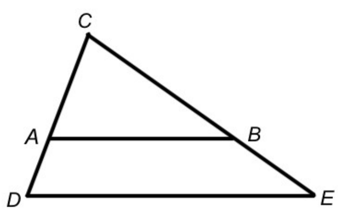
1. $CB = 8$, $BE = 4$ and $AD = 3$. Find AC.



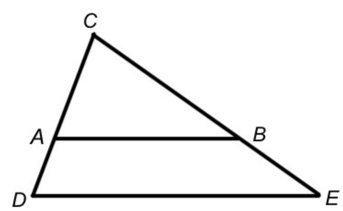
2. $CA = 6$, $AD = 2$ and $BE = 5$. Find CE.



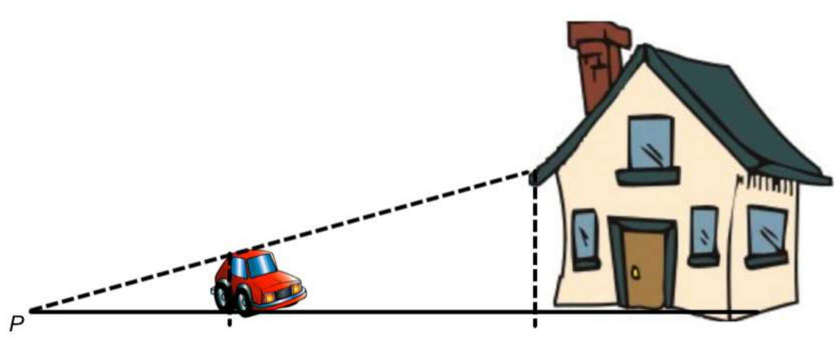
3. $CA = 4$, $AD = 2$ and $AB = 8$. Find DE.



4. $CA = 3$, $AD = 2$ and $DE = 15$. Find AB.



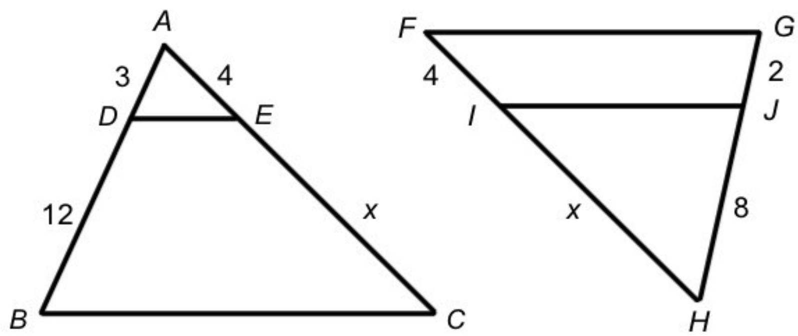
5. Kolby needs to fix a leaky roof on his mom's house but doesn't own a ladder. He thinks that a 25-foot ladder will be long enough to reach the roof, but he needs to be sure before he spends the money to buy one. He chooses a point *P* on the ground where he can visually align the roof of his 4.25 ft tall car with the edge of the roof of the house. If point *P* is 8.5 ft from the car and the car is 23 ft from the house, will the 25-foot ladder be tall enough?



Lesson 3: Applying the Triangle Side Splitter Theorem II

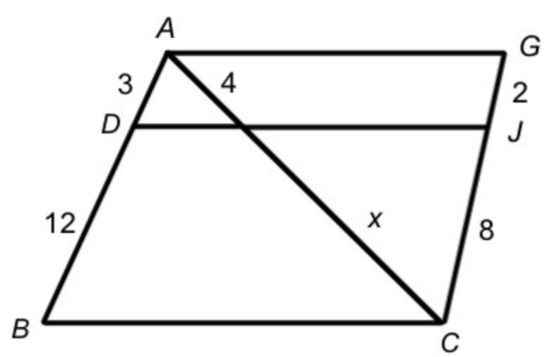
Opening Exercise

In the two triangles pictured below, $\overline{DE} \parallel \overline{BC}$ and $\overline{FG} \parallel \overline{IJ}$. Find the measure of *x* in both triangles.



What is the relationship between ΔABC and ΔFGH?

Since the two triangles share a common side, look what happens when we push them together:

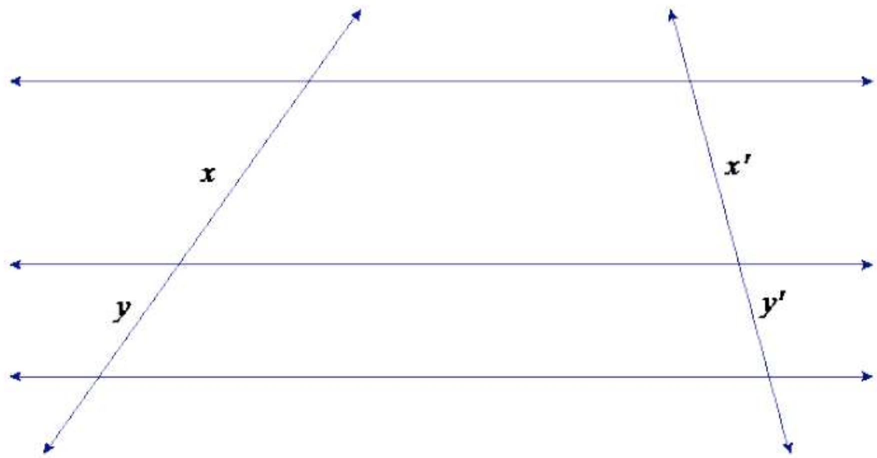


We now have 3 parallel lines cut by 3 transversals. Is the transversal on the left in proportion to the transversal on the right?

Example 1

We are going to do an informal proof of the following theorem:

Theorem: *If 3 or more parallel lines are cut by 2 transversals, then the segments of the transversals are in proportion.*



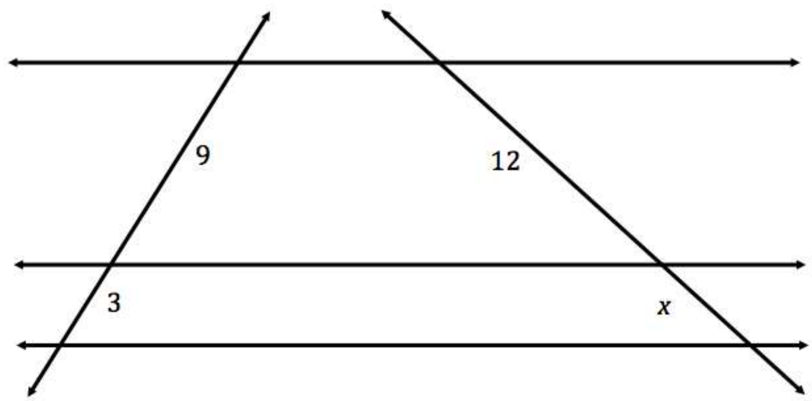
By drawing an auxiliary line to create two similar triangles that share a common side, we will show:

$$\frac{x}{y} = \frac{x'}{y'}$$

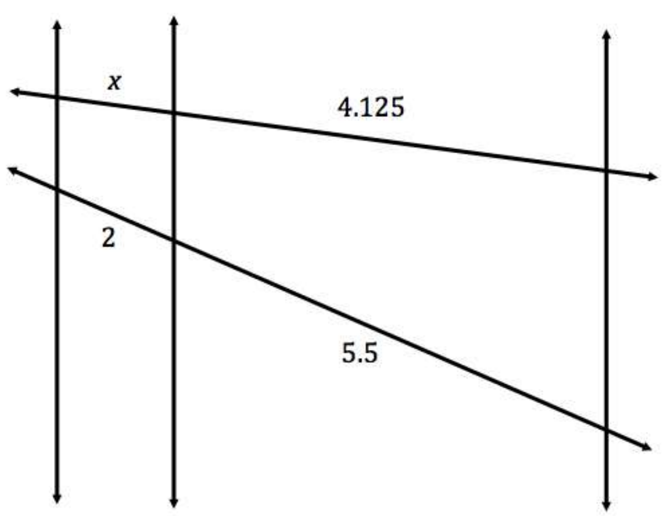
Exercises

In exercises 1 and 2, find the value of x. Lines that appear to be parallel are in fact parallel.

1.

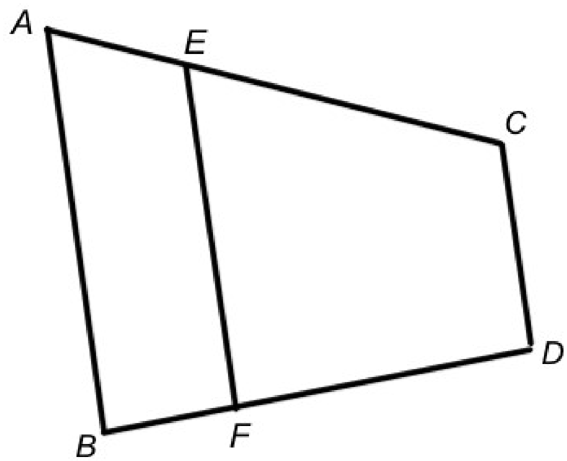


2.



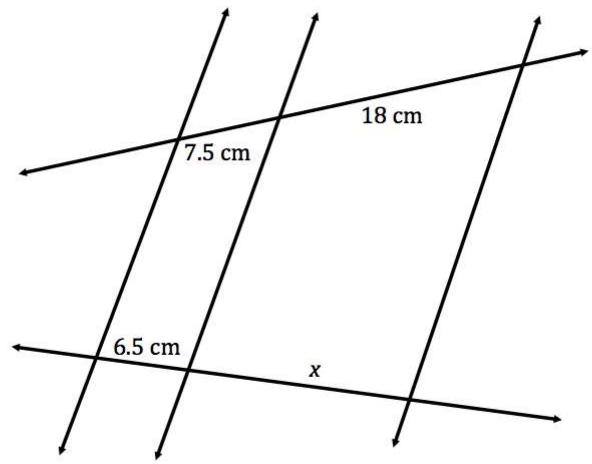
Example 2

In the diagram pictured below, $AB \parallel EF \parallel CD$, AB = 20, CD = 8, FD = 12 and $AE : EC = 1 : 3$. If the perimeter of trapezoid $ABDC$ is 64, find AE and EC.

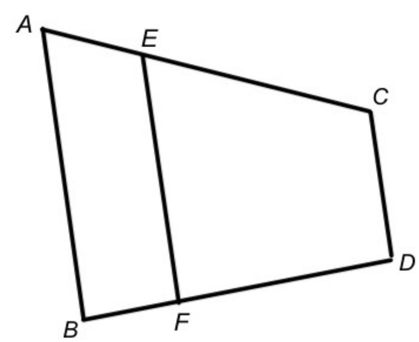


Homework

1. Find the value of x. Lines that appear to be parallel are in fact parallel.



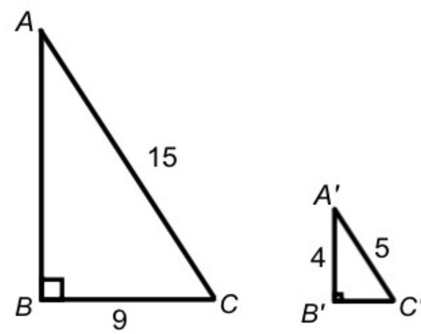
2. In the diagram pictured below, $AB \parallel EF \parallel CD$, AB = 7, CD = 4, FD = 6 and $AE : EC = 1:4$. If the perimeter of trapezoid $ABDC$ is 31, find AE and EC.



Lesson 4: Properties of Similar Triangles

Opening Exercise

Given $\Delta ABC \sim \Delta A'B'C'$ pictured to the right:



Find the lengths of the missing sides.

Find the perimeter of the triangles.

Find the area of the triangles.

Exercise 1

Using the opening exercise, answer the following:

a. What is the relationship between the sides of the triangles and the perimeters?

b. What is the relationship between the sides of the triangles and the areas?

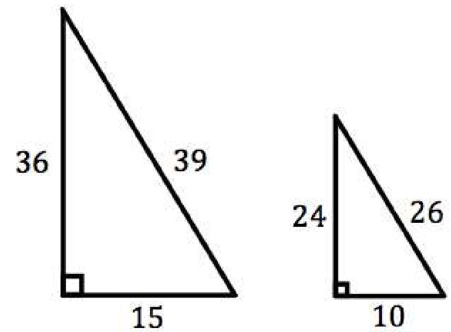
c. Make a hypothesis of these relationships.

Example 2

Let's test the hypothesis we made in Exercise 1.

Given the similar triangles pictured to the right, find:

a. Ratio of the sides

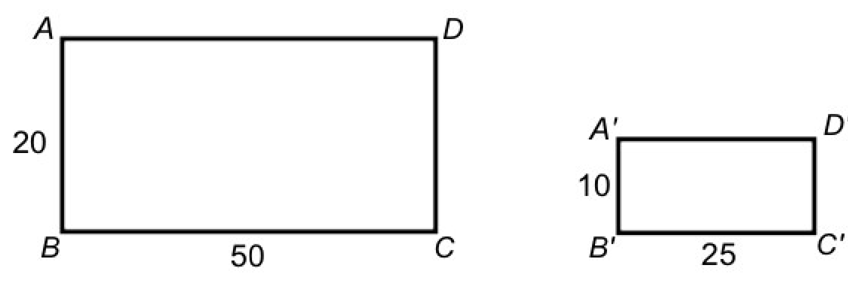


b. Ratio of the perimeters

c. Ratio of the areas

Given the similar rectangles pictured to the right, find:

a. Ratio of the sides



b. Ratio of the perimeters

c. Ratio of the areas

CONCLUSIONS:

When two figures are similar and the ratio of their sides is a:b, then:

The perimeters are in the ratio of:

The areas are in the ratio of:

If the similar figures were 3-dimensional, what do you think is the relationship between the volumes of the figures?

Exercises

1. Two triangles are similar. The sides of the smaller triangle are 6,4,8. If the shortest side of the larger triangle is 6, find the length of the longest side.

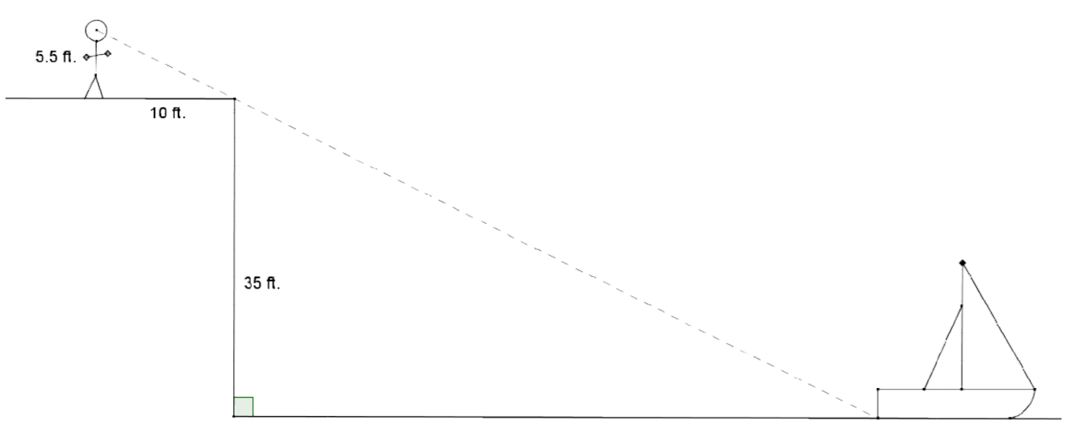
2. The sides of a triangle are 8, 5, and 7. If the longest side of a similar triangle measures 24, find the perimeter of the larger triangle.

3. The sides of a triangle are 7, 8 and 10. What is the length of the shortest side of a similar triangle whose perimeter is 75?

4. Find the ratio of the areas of two similar triangles in which the ratio of the pair of corresponding sides is 3:2.

5. Find the ratio of the lengths of a pair of corresponding sides in two similar polygons if the ratio of the areas is 4:25.

6. Caterina's boat has come untied and floated away on the lake. She is standing atop a cliff that is 35 feet above the water in a lake. If she stands 10 feet from the edge of the cliff, she can visually align the top of the cliff with the water at the back of her boat. Her eye level is 5.5 feet above the ground. How far out from the cliff, to the *nearest tenth*, is Catarina's boat?



Homework

1. Two triangles are similar. The sides of one triangle are 4,8 and 10. If the shortest side of the second triangle is 12, find the length of the missing sides.

2. Two triangles are similar. The sides of the smaller triangle are 6, 4, and 8. The perimeter of the larger similar triangle is 27. Find the length of the shortest side of the larger triangle.

3. Find the ratio of the areas of two similar triangles in which the ratio of the pair of corresponding sides is 5:3.

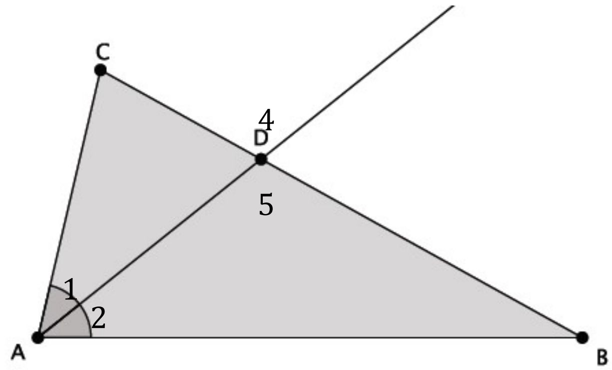
4. Find the ratio of the volumes of two similar triangles in which the ratio of the pair of corresponding areas is 9:16.

Lesson 5: The Angle Bisector Theorem

Opening Exercise

The Angle Bisector Theorem states:

In ΔABC, if the angle bisector of $\angle A$ meets side BC at point D, then BD:CD = BA:CA.



We are going to fill in the missing reasons of the proof of the Angle Bisector Theorem.

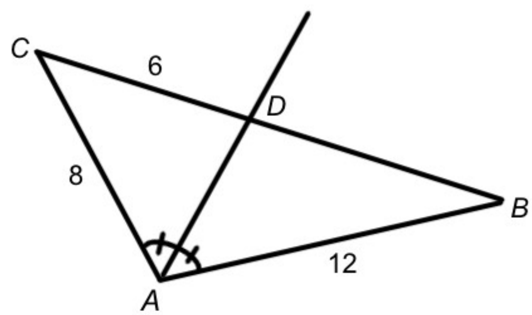
Given: AD is the angle bisector of $\angle A$

Prove: $\frac{BD}{CD} = \frac{BA}{CA}$

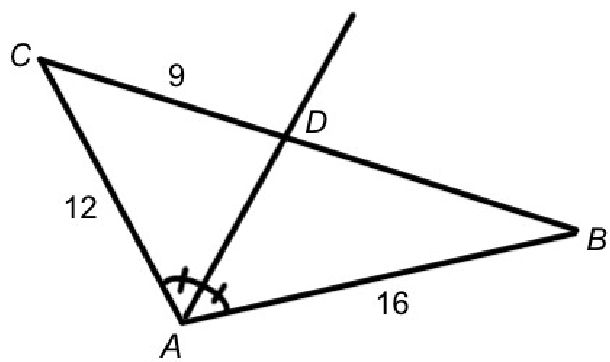
Statements	Reasons
1. AD is the angle bisector of $\angle A$	1.
2. $\angle 1 \cong \angle 2$	2.
3. Draw $CE \parallel$ to AB where E is the point of intersection of AD. (Label $\angle CED$ as $\angle 3$)	3.
4. $\angle 2 \cong \angle 3$	4.
5. $\angle 4 \cong \angle 5$	5.
6. ΔCDE ΔBDA	6.
7. $\frac{BD}{CD} = \frac{BA}{CE}$	7.
8. $\angle 1 \cong \angle 3$	8.
9. ΔACE is isosceles	9.
10. $CA \cong CE$	10.
11. $\frac{BD}{CD} = \frac{BA}{CA}$	11.

Exercises

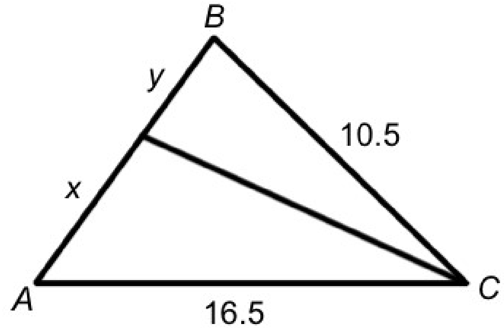
1. In $\triangle ABC$ pictured below, AD is the angle bisector of $\angle A$. If CD = 6, CA = 8 and AB = 12, find BD.



2. In $\triangle ABC$ pictured below, AD is the angle bisector of $\angle A$. If CD = 9, CA = 12 and AB = 16, find BD.

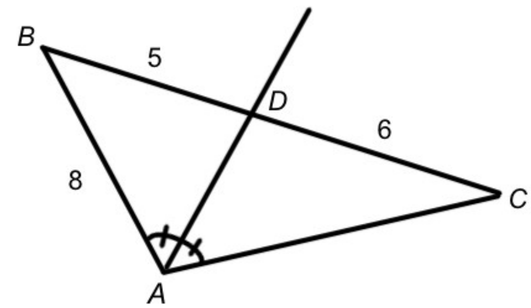


3. The sides of $\triangle ABC$ pictured below are 10.5, 16.5 and 9. An angle bisector meets the side length of 9. Find the lengths of x and y.



Homework

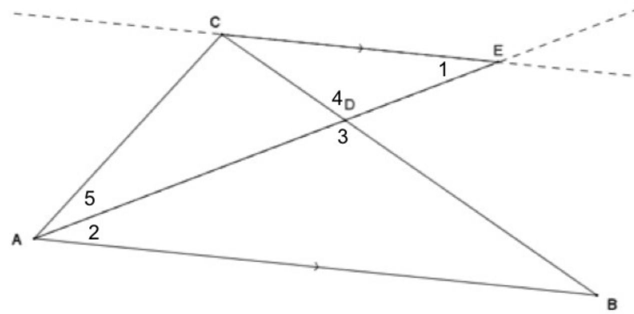
1. In $\triangle ABC$ pictured, AD is the angle bisector of $\angle A$. If $BD = 5$, $CD = 6$ and $AB = 8$, find AC.



2. The converse of the Angle Bisector Theorem states:

In $\triangle ABC$, if AE meets BC at D such that BD:CD = BA:CA, then AD is the angle bisector of $\angle A$.

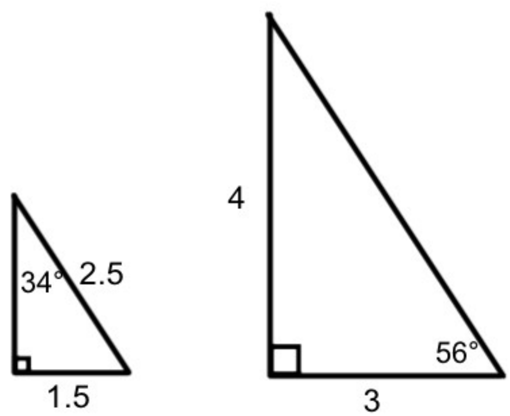
Note: $AB \| CE$



	Statements		**Reasons**
1.	$\frac{BD}{CD} = \frac{BA}{CA}$	1.	Given
2.	$\angle 1 \cong \angle 2$	2.	Alt. interior $\angle's$ are $\cong$
3.	$\angle 3 \cong \angle 4$	3.	
4.	$\triangle CDE \sim \triangle BDA$	4.	
5.	$\frac{BD}{CD} = \frac{BA}{CE}$	5.	
6.	$CE \cong CA$	6.	The given and step 5 imply this
7.	$\triangle ACE$ is isosceles	7.	
8.	$\angle 1 \cong \angle 5$	8.	
9.	$\angle 2 \cong \angle 5$	9.	Substitution
10.	AD is the angle bisector of $\angle A$	10.	

Lesson 6: Special Relationships Within Right Triangles

Opening Exercise



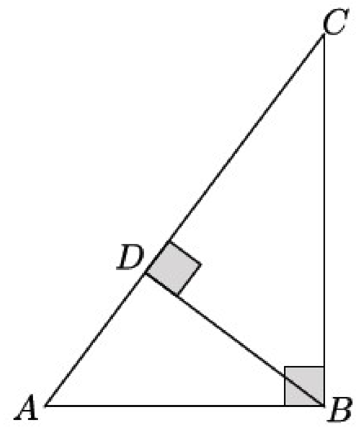
Given the triangles pictured above, answer the following:

a. Are the triangles similar? Explain.

b. Find all of the missing sides and angles.

Example 1

In ΔABC pictured to the right, $\angle B$ is a right angle and BD is the altitude.



a. How many triangles do you see in the picture?

b. Identify the triangles by name.

c. Is the big triangle similar to the small triangle? Explain.

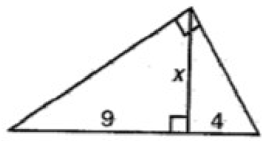
d. Is the big triangle similar to the medium triangle? Explain.

e. What is the relationship between the small triangle and the medium triangle? How do you know?

Exercises

Find the value of *x*:

1.



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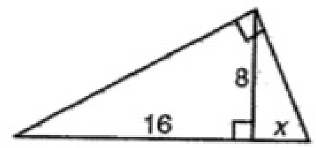

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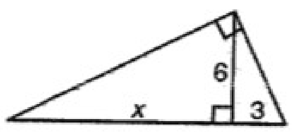

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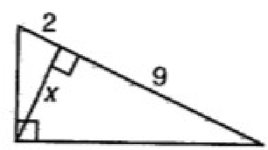

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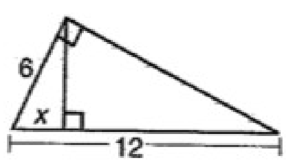
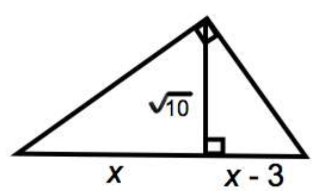
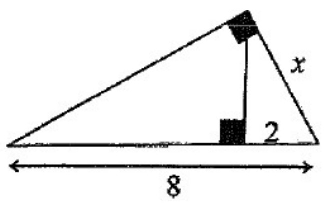


6.

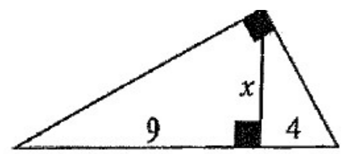


Homework

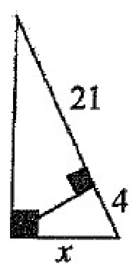
1.



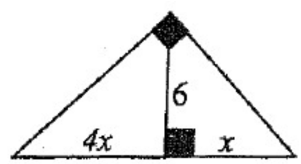
2.



3.



4.



Lesson 7: Simplifying Radicals

Opening Exercise

a. Write the products in column 2.

Factors	Products
$1 \cdot 1$ or 1^2	
$2 \cdot 2$ or 2^2	
$3 \cdot 3$ or 3^2	
$4 \cdot 4$ or	
$5 \cdot 5$ or	
$6 \cdot 6$ or	
$7 \cdot 7$ or	
$8 \cdot 8$ or	
$9 \cdot 9$ or	
$10 \cdot 10$ or	
$11 \cdot 11$ or	
$12 \cdot 12$ or	
$13 \cdot 13$ or	
$14 \cdot 14$ or	
$15 \cdot 15$ or	

b. What is another name for the products in Column 2?

c. Now let's look at variables:

Factors	Products
$x \cdot x$	
$x^2 \cdot x^2$	
$x^3 \cdot x^3$	
$x^4 \cdot x^4$	
$x^5 \cdot x^5$	

Example 1

Simplify the square root of each perfect square from column 1.

$\sqrt{\text{Perfect Squares}}$	**Simplified Answer**
$\sqrt{1}$	
$\sqrt{4}$	
$\sqrt{9}$	
$\sqrt{16}$	
$\sqrt{25}$	
$\sqrt{36}$	
$\sqrt{49}$	
$\sqrt{64}$	
$\sqrt{81}$	
$\sqrt{100}$	
$\sqrt{121}$	
$\sqrt{144}$	

Simplify each perfect square variable:

$\sqrt{\text{Perfect Squares}}$	**Simplified Answer**
$\sqrt{x^2}$	
$\sqrt{x^4}$	
$\sqrt{x^6}$	
$\sqrt{x^8}$	

Simplify the following:

a. $\sqrt{64x^2}$

b. $\sqrt{2500x^{1000}}$

c. $\sqrt{\dfrac{4}{x^4}}$

Example 2

Simplifying *NON-PERFECT* squares:

$$\sqrt{\text{Non-Perfect Square}} =$$

$$\sqrt{Largest\ perfect\ square} \bullet \sqrt{remaining\ factor}$$

Simplify the following:

a. $\sqrt{32}$

b. $\sqrt{500}$

c. $\sqrt{16x^3}$

d. $\sqrt{3600x^5}$

e. $\sqrt{48x^3}$

f. $\sqrt{125}$

g. $3\sqrt{45x^2}$

h. $2\sqrt{52}$

j. $16\sqrt{16}$

Homework

In 1-5, simplify the given expressions:

1. $\sqrt{144}$

2. $3\sqrt{72}$

3. $3\sqrt{32x^2}$

4. $4\sqrt{16x^3}$

5. $\sqrt{1000x^{1000}}$

Lesson 8: Multiplying and Dividing Radicals

Opening Exercise

Simplify each of the following:

$\sqrt{3600x^{500}}$ $\sqrt{25x^{3}}$ $\sqrt{98}$

Example 1

Multiplying and Dividing Radicals:

1. Multiply/Divide the Coefficient
2. Multiply/Divide the Radicand (number under the radical)
3. Simplify

Simplify the following:

a. $(\sqrt{6})\cdot(\sqrt{60})$

b. $\frac{\sqrt{96}}{\sqrt{4}}$

Exercises

Simplify:

1. $\sqrt{\frac{17}{25}}$

2. $(\sqrt{x})(\sqrt{4x})$

3. $\left(4\sqrt{30}\right)\left(6\sqrt{3}\right)$

4. $\frac{16\sqrt{20}}{2\sqrt{5}}$

Rationalizing Denominators

(you are NOT allowed to have a radical in the denominator)

1. Check to see if you can divide the numerator and denominator to eliminate the radical.
2. If you CANNOT eliminate the radical in the denominator, you must *rationalize* the denominator.
3. Multiple the numerator and denominator by the radical in the denominator.
4. Simplify the numerator and denominator.

Example 2

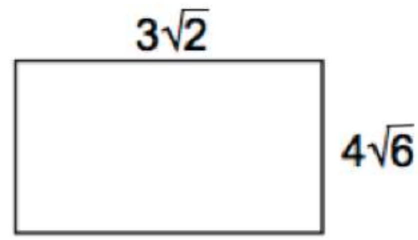
Rationalize the following:

1. $\frac{1}{\sqrt{2}}$

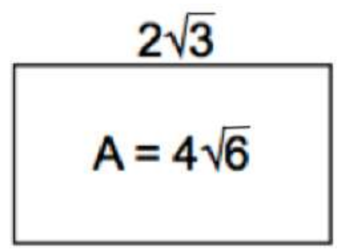
2. $\sqrt{\frac{7}{5}}$

Exercises

1. Find the area of the rectangle in simplest radical form.



2. Find the width of the rectangle with an area of $4\sqrt{6}$ in simplest radical form.



Homework

In 1-6, simplify the given expressions:

1. $\sqrt{52}$

2. $\sqrt{48x^6}$

3. $\left(5\sqrt{3}\right)\left(7\sqrt{2}\right)$

4. $\left(3\sqrt{2}\right)\left(\sqrt{14}\right)$

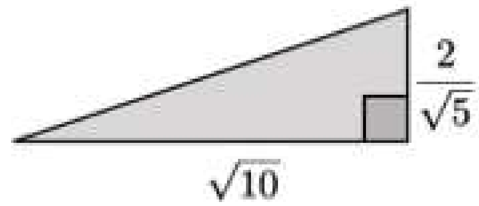
5. $\dfrac{10\sqrt{6}}{5\sqrt{2}}$

6. $\dfrac{2}{\sqrt{3}}$

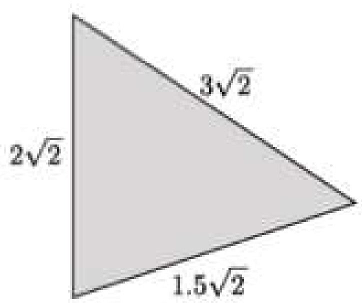
Lesson 9: Adding and Subtracting Radicals

Opening Exercise

Calculate the area of the triangle:



The triangle shown below has a perimeter of $6.5\sqrt{2}$ units. Make a conjecture about how this answer was reached.



Example 1

Simplify the following expressions:

a. $3\sqrt{18}+10\sqrt{2}$

b. $19\sqrt{2}-2\sqrt{8}$

Adding and Subtracting Radicals:

1. Simplify all terms
2. Add/subtract the coefficients of like radicals
3. Keep the radical

Exercises:

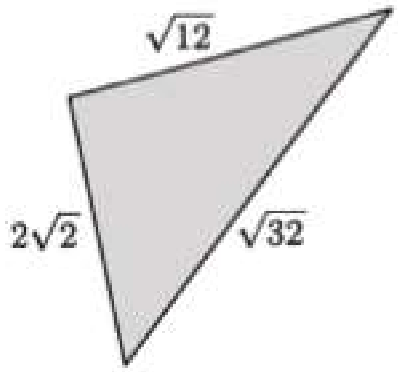
Simplify the following expressions:

1. $18\sqrt{5}-12\sqrt{5}$

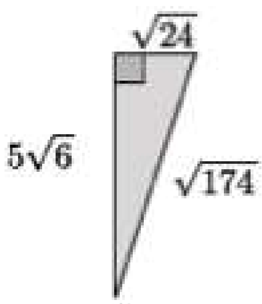
2. $\sqrt{24}+5\sqrt{24}$

3. $2\sqrt{7}+4\sqrt{63}$

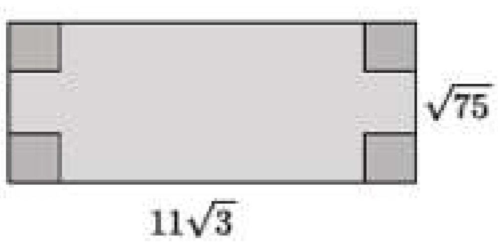
4. What is the perimeter of the triangle shown below?



5. Determine the area of the triangle shown. Simplify as much as possible.



6. Determine the area and perimeter of the rectangle shown. Simplify as much as possible



Homework

Simplify the following expressions:

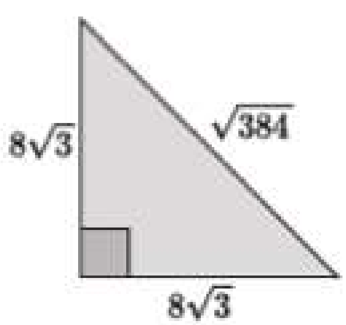
1. $5\sqrt{28}$

2. $2\sqrt{3}+5\sqrt{3}$

3. $\sqrt{8}+\sqrt{18}-\sqrt{2}$

4. $\dfrac{\sqrt{50}+\sqrt{98}}{6}$

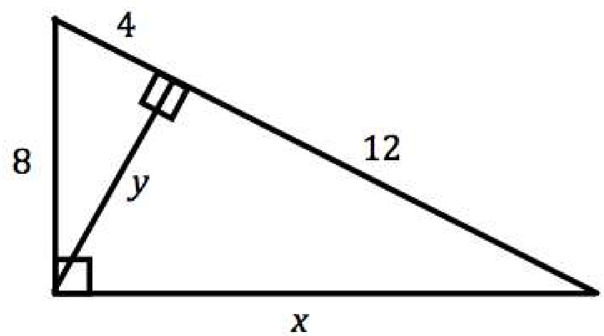
5. Determine the area and perimeter of the triangle shown. Simplify as much as possible.



Lesson 10: Proof of Pythagorean Theorem

Opening Exercise

In the triangle pictured below, find *x* and *y*:



*****There is another method that can be used to find *x* and *y*.******

Pythagorean Theorem:

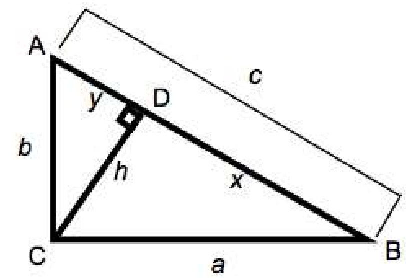
$$a^2 + b^2 = c^2$$

Use to find the missing side of a right triangle when 2 sides are given.

Example 1

We are going to prove the Pythagorean Theorem!

Use the following picture to separate out the “small”, “medium” and “large” triangles. Label each triangle with the appropriate vertex and side labels.



“small” “large” “medium”

Based on what we did in Lesson 6, we know that all 3 triangles are similar, so we can set up proportions.

a. Which two triangles share $\angle B$?

b. Write a proportion that compares these two triangles using the ratio *longer leg:hypotenuse*, then perform cross products.

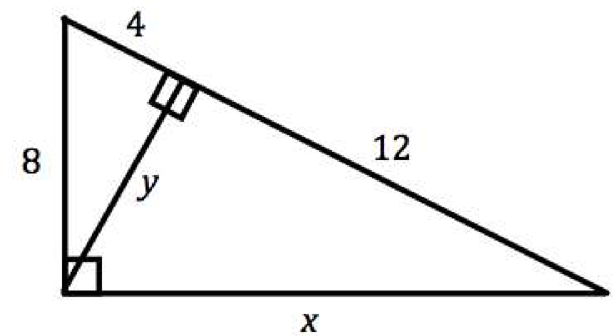
c. What two triangles share $\angle A$?

d. Write a proportion that compares these two triangles using the ratio *shorter leg:hypotenuse*, then perform cross products.

e. Our goal is to show that $a^2 + b^2 = c^2$. Let’s use substitution and our cross products.

Example 2

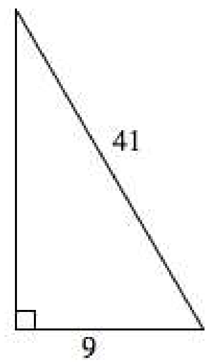
Let's go back to the opening exercise to find *x* and *y* using Pythagorean Theorem.



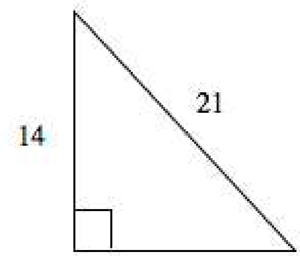
Exercises

1. Find the missing sides, to the *nearest tenth*:

a.



b.



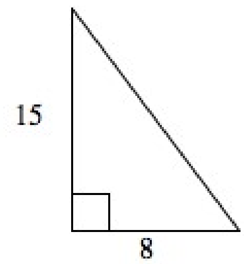
2. Tanya runs diagonally across a rectangular field that has a length of 40 yards and a width of 30 yards. What is the length of the diagonal, in yards, that Tanya runs?

3. The sides of a triangle are 7, 11, and 18. Do these sides form a right triangle? Justify your answer.

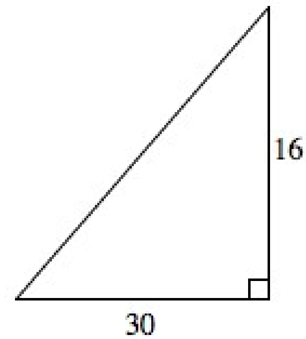
Homework

1. Find the missing sides:

a.



b.



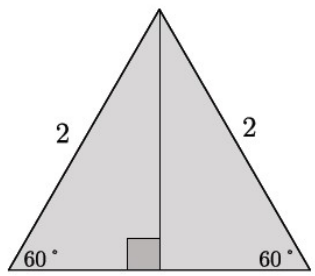
2. A 12-foot ladder leans against a house. The foot of the ladder is 7 feet away from the house. How high up the house does the ladder reach, to the *nearest tenth*?

3. A triangle has sides 20, 21 and 29. Is it a right triangle? Justify your answer.

Lesson 11: Special Right Triangles

Opening Exercise

An equilateral triangle has sides of length 2 and angle measures of 60, as shown below. The altitude from one vertex to the opposite side divides the triangle into two right triangles.



a. Are the two triangles congruent? Explain.

b. What is the length of the shorter leg of each of the right triangles? Explain.

c. Use the Pythagorean theorem to determine the length of the altitude.

d. Write the ratio that represents $shorter\ leg: hypotenuse$.

e. Write the ratio that represents $longer\ leg: hypotenuse$.

f. Write the ratio that represents $shorter\ leg: longer\ leg$.

Summary of the 30-60-90 Right Triangle Ratios

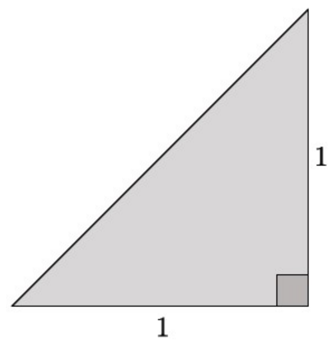
Example 1

By the AA criterion, any triangle with measures 30–60–90 will be similar to this triangle. If a 30–60–90 triangle has a hypotenuse of length 16, what are the lengths of the legs?

If a 30–60–90 triangle, the short leg has a length of 5. What are the lengths of the other leg and the hypotenuse?

Example 2

An isosceles right triangle has leg lengths of 1, as shown.



a. What are the measures of the other two angles? Explain.

b. Use the Pythagorean Theorem to determine the length of the hypotenuse of the right triangle.

c. Is it necessary to write all three ratios: $shorter\ leg: hypotenuse$, $longer\ leg: hypotenuse$, and $shorter\ leg: longer\ leg$? Explain.

d. Write the ratio that represents $leg: hypotenuse$.

Summary of the 45-45-90 Right Triangle Ratios

Example 3

By the AA criterion, any triangle with measures 45–45–90 will be similar to this triangle. If a 45–45–90 triangle has a hypotenuse of length 20, what are the lengths of the legs?

If a 45–45–90 triangle has a leg length of 20, what are the lengths of the other leg and the hypotenuse?