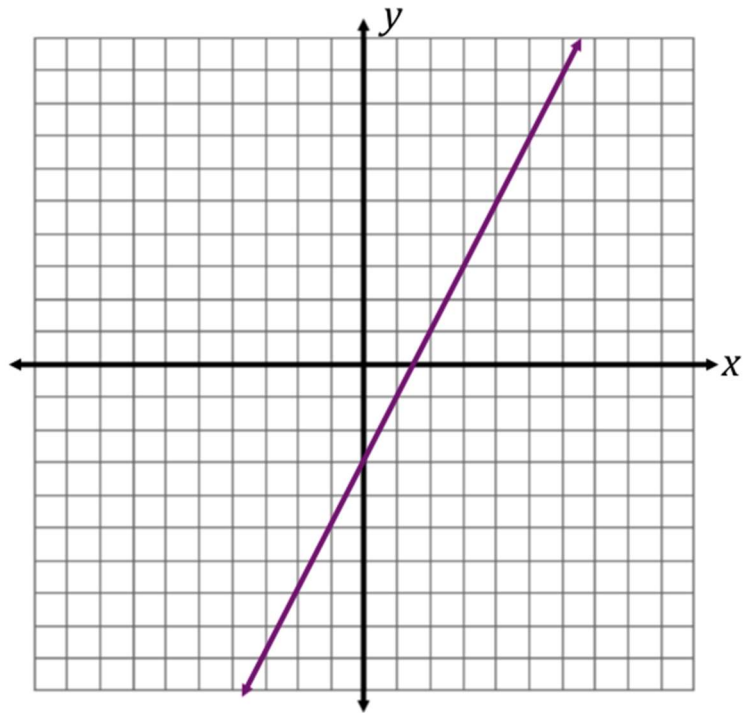


# Unit 8

## Connecting Algebra and Geometry Through Coordinates

### Lesson 1: Graphing Linear Inequalities

#### Opening Exercise



Using the graph pictured above, answer the following:

- What are the coordinates of the  $y$ -intercept?
- What is the slope of the line?
- Write the equation of the line.
- On the grid above, draw a line parallel to the given line.
- Write the equation of the line you drew in part  $d$ .

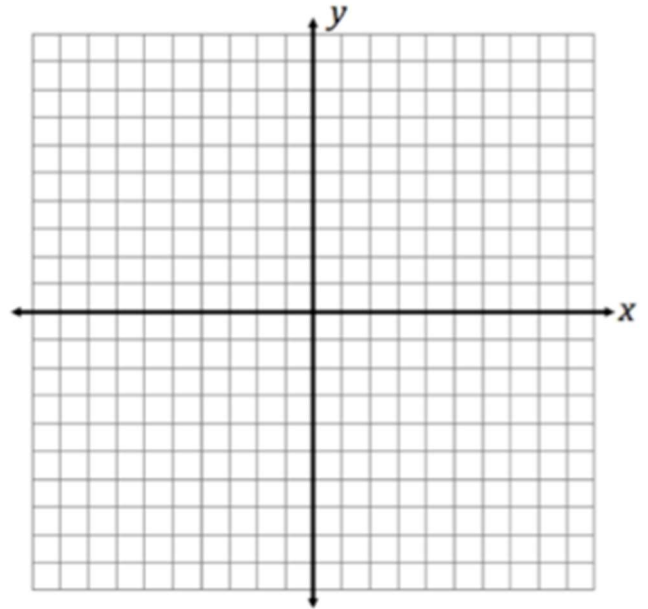


### Example 1

Solve the following system of equations graphically:

$$y = -3x + 8$$

$$y + 2 = \frac{1}{3}x$$



### Graphing Linear Inequalities

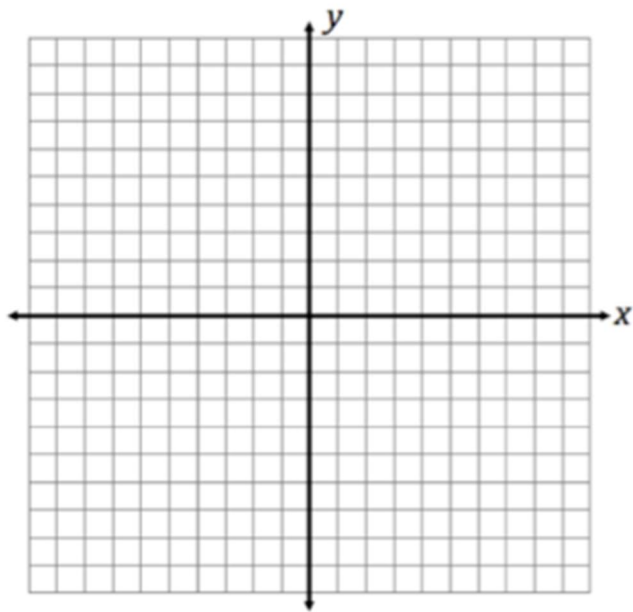
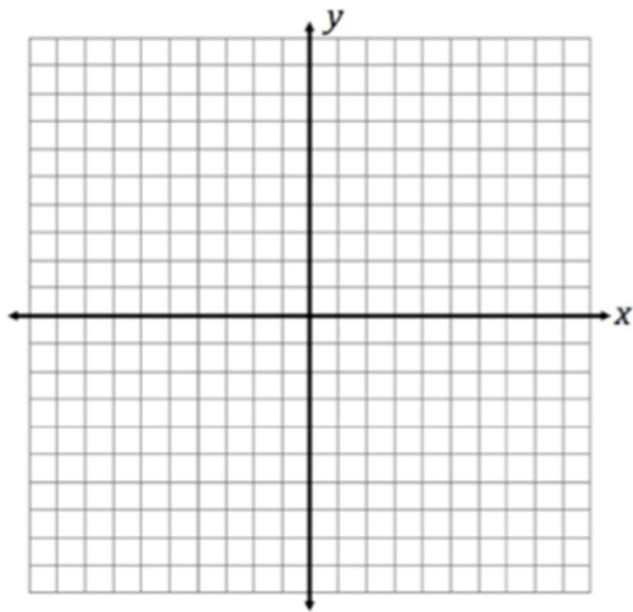
	Type of Line	Shading
$<$		
$>$		
$\leq$		
$\geq$		

### Example 2

Graph the following linear inequalities on the grids provided:

a.  $x < 2$

b.  $y \geq \frac{3}{2}x - 5$



### Example 3

Graph the following system of inequalities:

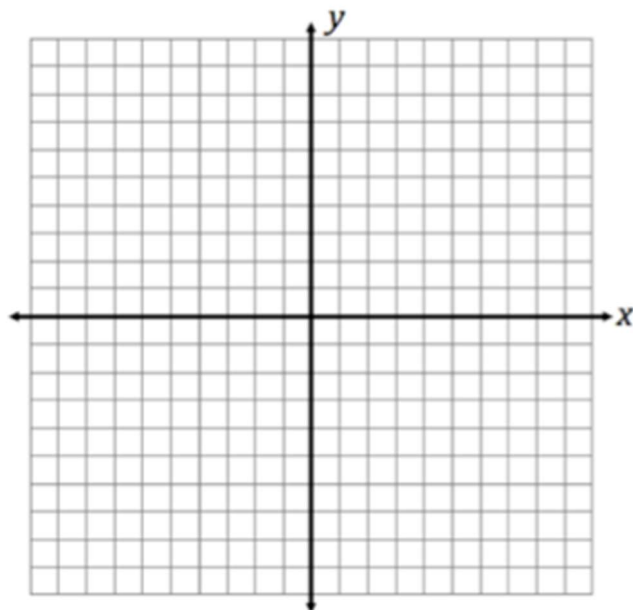
$$y < 2x + 1$$

$$y \geq -3x - 2$$

a. Is  $(-2, 4)$  in the solution set?

b. Is  $(1, 3)$  in the solution set?

c. The region is the intersection of how many half-planes? Explain how you know.

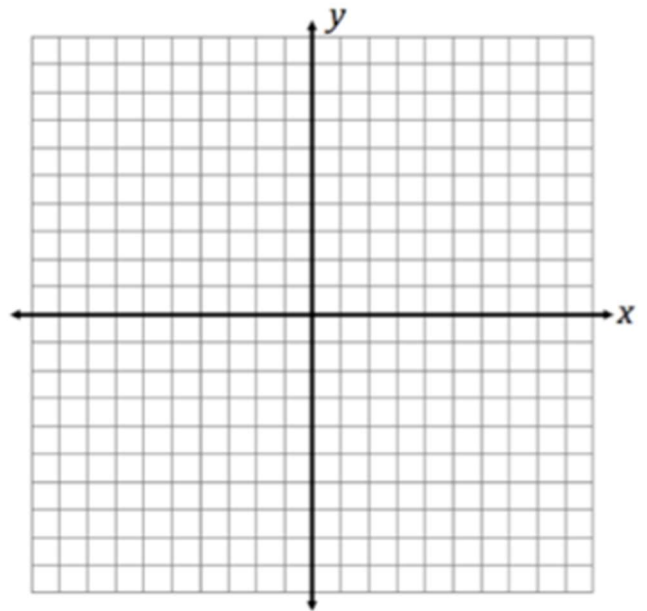


## Homework

1. Solve the following system of equations graphically:

$$y = -2x - 3$$

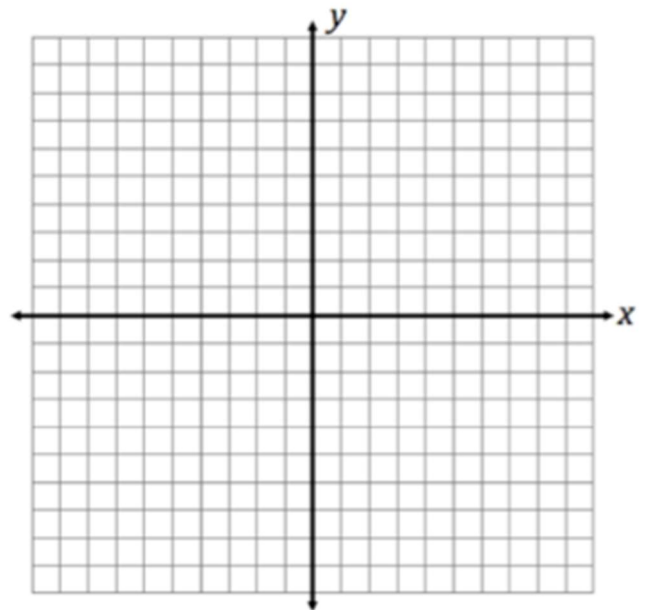
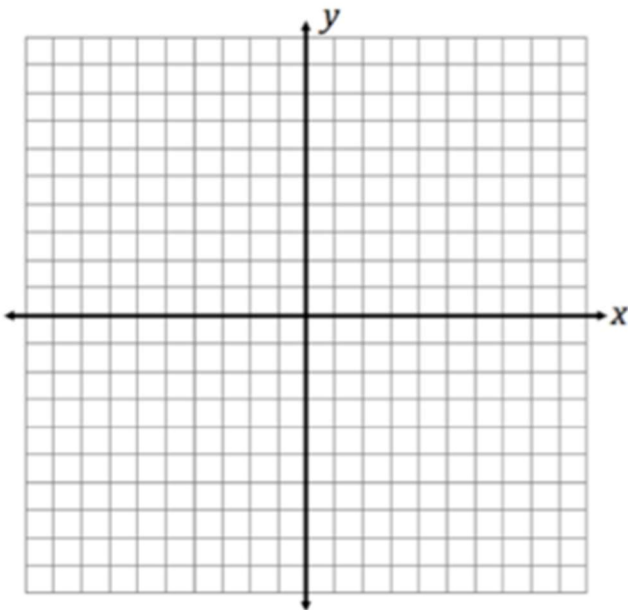
$$y = \frac{1}{2}x + 2$$



2. Graph the following inequalities:

a.  $y > -3$

b.  $y \leq -\frac{2}{3}x + 4$



## Lesson 2: Graphing Systems of Linear Inequalities

### Opening Exercise

Graph the following system of inequalities:

$$y \leq -2$$

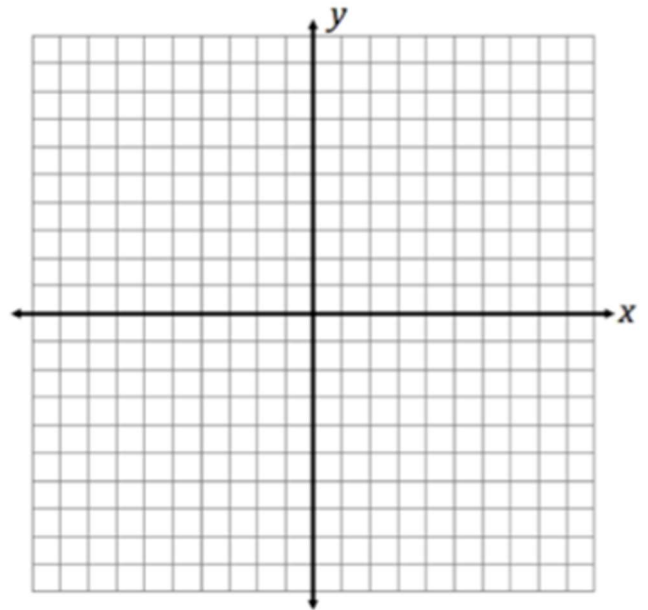
$$x > 3$$

a. Is  $(5, -3)$  a solution? Explain.

b. Is  $(4, -2)$  a solution? Explain.

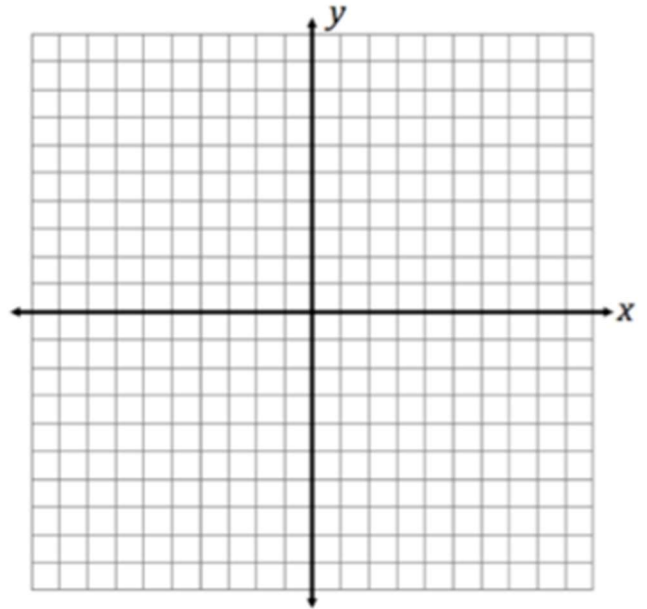
c. Is  $(3, -5)$  a solution? Explain.

d. The region is the intersection of how many half-planes? Explain how you know.



### Example 1

- a. Graph the equation  $y = -2x + 6$ .
- b. Find the area of the triangle bounded by the given line, the  $x$ -axis, and the  $y$ -axis. Shade this shape.
- c. Write three inequalities to represent the shaded region.



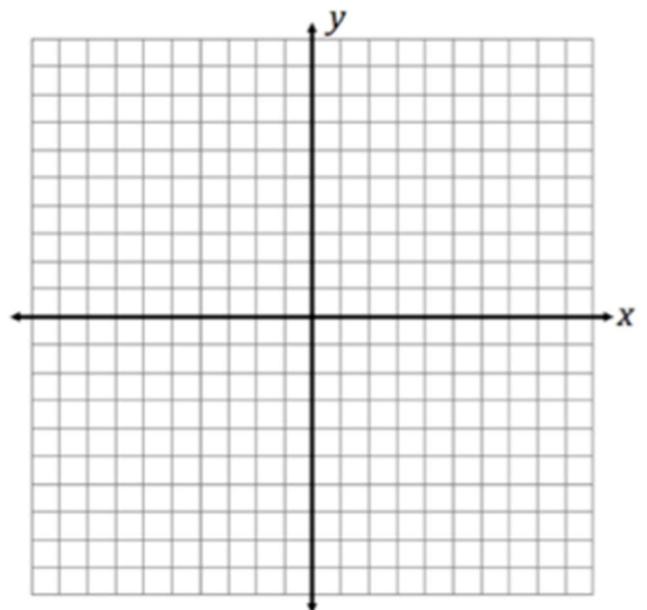
### Example 2

Draw the triangular region in the plane described by the following system of inequalities:

$$y \leq 3x$$

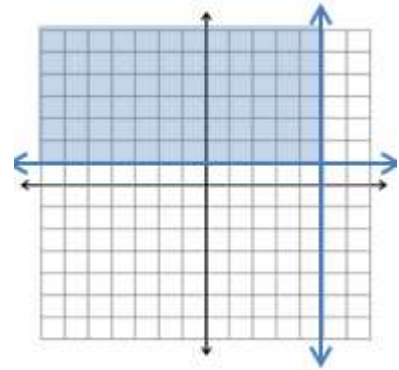
$$y \geq \frac{1}{2}x$$

$$y \leq -2x + 5$$



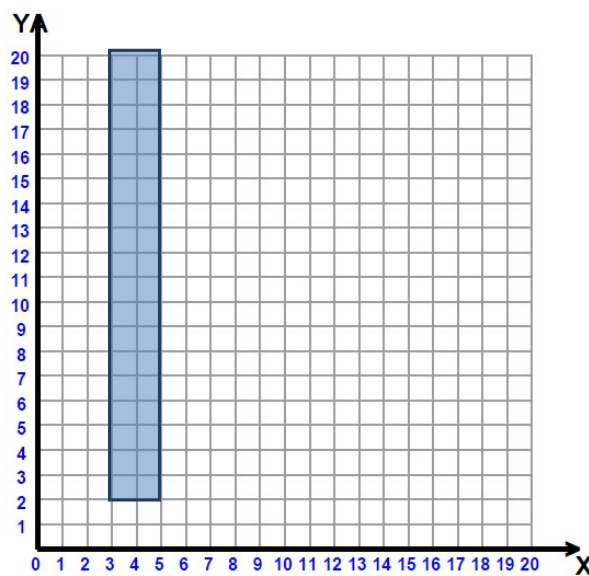
### Example 3

- Write the system of inequalities that describes the infinite shaded region:
- Describe the region in words
- How many half-planes intersect to form this region?



### Example 4

Write the system of inequalities that describes the finite shaded region and determine how many half-planes form this region.

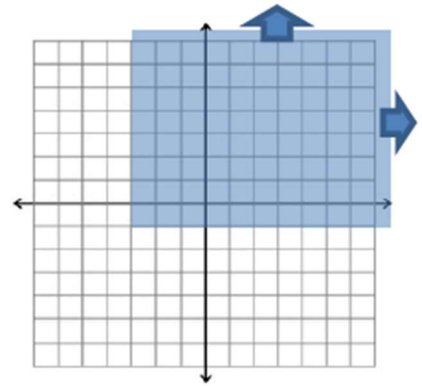




## Homework

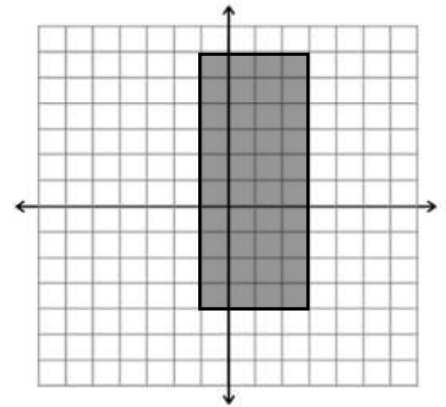
1. Given the region shown:

- How many half-planes intersect to form this region?
- Name three points on the boundary of the region.
- Describe the region in words.

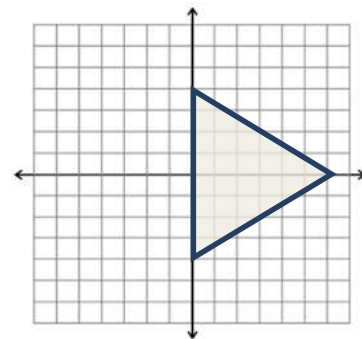


2. Given the region shown:

- Write the coordinates of the vertices.
- Write the system of inequalities that describes the region.
- What is the length of the diagonal to the *nearest hundredth*?
- Give the coordinates of a point that is both in the region and on one of the diagonals.



3. Given the triangular region shown, describe this region with a system of inequalities:

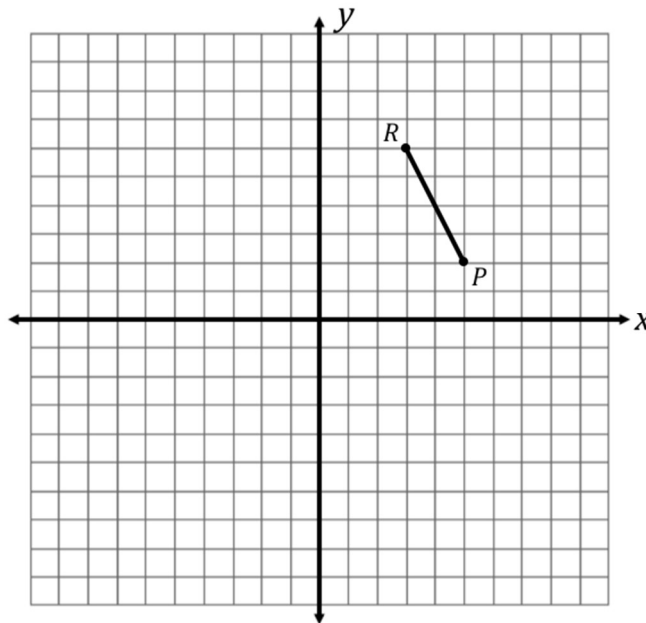


## Lesson 3: Parallel and Perpendicular Lines

### Opening Exercise

Line segment  $\overline{PR}$  is pictured connecting points  $P(5, 2)$  and  $R(3, 6)$ .

1. Graph  $\overline{P'R'}$ , the image of  $\overline{PR}$  after  $R_{0,90^\circ}$  and state the endpoints.
2. Graph  $\overline{P''R''}$ , the image of  $\overline{PR}$  after  $R_{0,180^\circ}$  and state the endpoints.
3. Graph  $\overline{P'''R'''}$ , the image of  $\overline{PR}$  after  $R_{0,270^\circ}$  and state the endpoints.



### Example 1

Using the opening exercise, find the slope of the following segments:

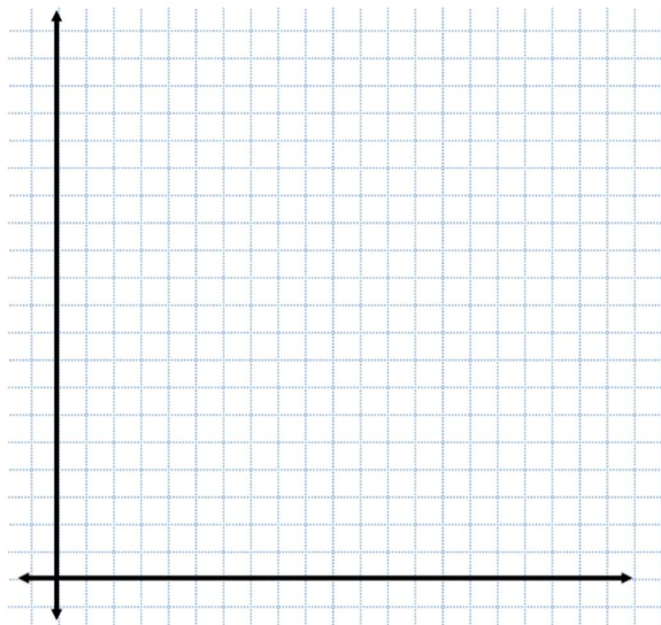
- a.  $\overline{PR}$
- b.  $\overline{P'R'}$
- c.  $\overline{P''R''}$
- d.  $\overline{P'''R'''}$

What do you notice about the slopes?

## Example 2

The line segment connecting  $(3, 7)$  to  $(10, 1)$  is rotated counterclockwise  $90^\circ$  about the point  $(3, 7)$ .

- a. Plot the points and draw the line segment on the grid provided.
- b. Where will the rotated endpoint land? Connect  $(3, 7)$  to the new image.
- c. Determine the slope of the original segment.
- d. Determine the slope of the image.



- e. What do you notice about the slopes of the two line segments?
- f. Now rotate the original segment  $90^\circ$  clockwise. Before using a sketch, predict the coordinates of the rotated endpoint using what you know about the perpendicular slope of the rotated segment.

## Slopes of Parallel and Perpendicular Lines

When lines are parallel, their slopes are \_\_\_\_\_.

When lines are perpendicular, their slopes are \_\_\_\_\_.

### Exercises

1. Given the following pairs of equations, determine if the lines are parallel, perpendicular or neither. Explain your answer.

a.  $y = 2x - 5$   
 $y = \frac{1}{2}x + 7$

b.  $y = -3x + 1$   
 $y = \frac{1}{3}x$

c.  $5x + 3y = 2$   
 $9x - 15y = 1$

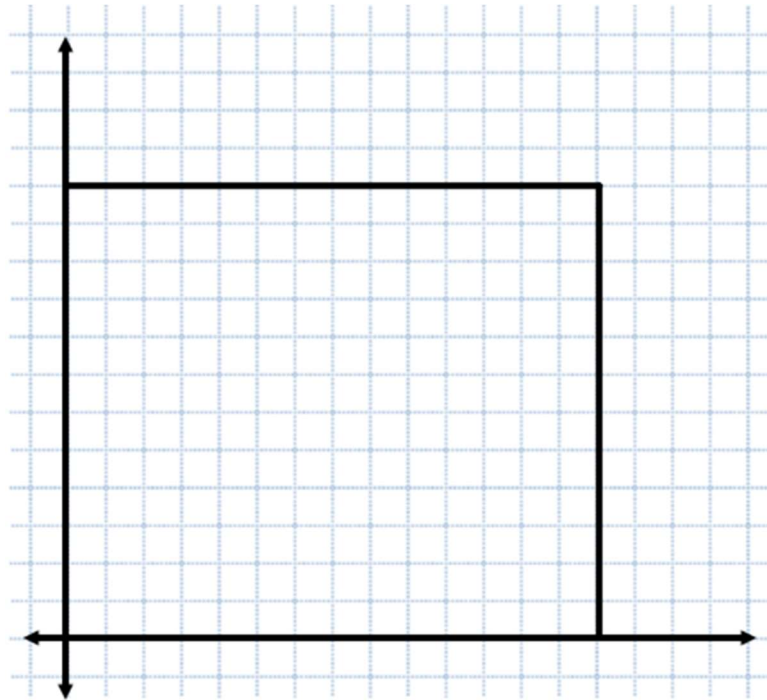
d.  $8x + 2y = 5$   
 $-4x - y = 9$

## Exercises

2. Given a line going through the points  $(8, -3)$  and  $(-4, -12)$ , determine the slope of the line that is:
  - a. Parallel to the given line
  
  
  
  
  
  
  
  
  
  
  - b. Perpendicular to the given line
  
3. Line  $p$  contains the points  $(8, 4)$  and  $(4, 3)$ . Line  $q$  contains the points  $(4, -9)$  and  $(2, -1)$ . Are the lines parallel, perpendicular or neither? How do you know?
  
  
  
  
  
  
  
  
  
  
4. Given the following slopes, what would be the slope of the perpendicular line:
  - a.  $m = 1$
  - b.  $m = 0$
  
  
  
  
  
  
  
  
  
  
5. Could a line going through the origin that is perpendicular to a line through the origin with a slope of  $\frac{1}{2}$  pass through the point  $(-1, 4)$ ? Explain.

## Homework

1. A robot vacuum is cleaning a 14 x 12 room as pictured. The robot starts in the southwest corner of the room (the origin) and travels along a line with a slope of 2. Every time the robot hits the wall it rotates  $90^\circ$  clockwise and continues along this new line until it hits the next wall. Graph the first 4 paths (lines) for the robot vacuum on the grid below and determine its coordinates before its fourth turn.



2. Are the following lines parallel, perpendicular or neither:  $6x - 3y = 7$   
Justify your answer.  $2x + 4y = 11$
3. A line through the origin has a slope of  $\frac{1}{3}$ . Carlos thinks the slope of a perpendicular line at the origin will be 3. Do you agree? Explain why or why not.



**Example 1**

Given  $A(5, -7)$  and  $B(8, 2)$ :

- a.* Find an equation for the line through  $A$  and perpendicular to  $\overline{AB}$ .
  
  
  
  
  
  
  
  
  
  
- b.* Find an equation for the line through  $B$  and perpendicular to  $\overline{AB}$ .

**Example 2**

Given points  $C(-4, 3)$  and  $D(3, 3)$ :

- a.* Write the equation of the line through  $C$  and perpendicular to  $\overline{CD}$ .
  
  
  
  
  
  
  
  
  
  
- b.* Write the equation of the line through  $D$  and perpendicular to  $\overline{CD}$ .



## Exercises

1. Write the equation of a line passing through the given point and parallel to the given line.

a.  $(2,1), y = 2x + 6$

b.  $(3,-2), 3x - 2y = 6$

2. Write the equation of a line passing through the given point and perpendicular to the given line.

a.  $(5, 3), y + 4x = 2$

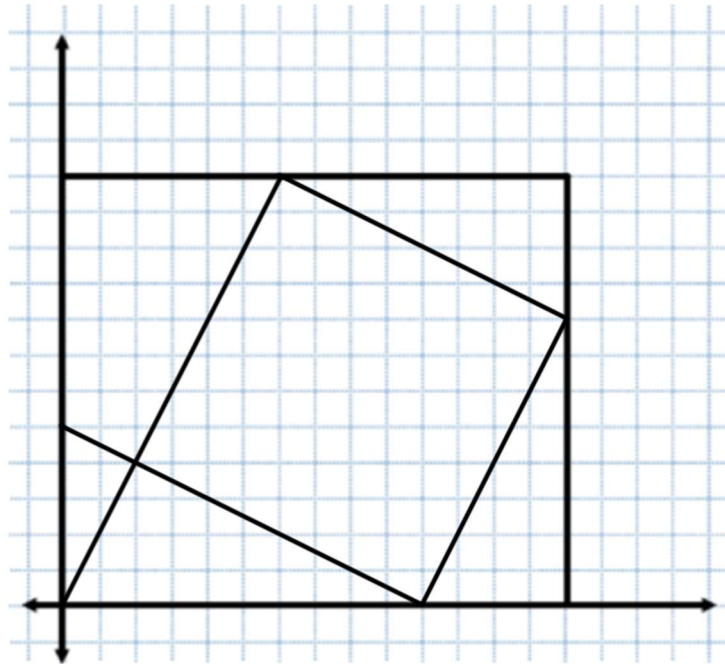
b.  $(10, 6), 5x + 3y = 4$

3. Our search robot is moving along the line  $y = 3x - 600$  and wishes to make a right turn at the point  $(400, 600)$ . Find an equation for the perpendicular line on which the robot is to move.

### Example 3

*From last night's homework:*

A robot vacuum is cleaning a 14 x 12 room as pictured. The robot starts in the southwest corner of the room (the origin) and travels along a line with a slope of 2. Every time the robot hits the wall it rotates  $90^\circ$  clockwise and continues along this new line until it hits the next wall. Graph the first 4 paths (lines) for the robot vacuum on the grid below.



Write the rectangular region formed by the first 4 paths of the robot as a system of inequalities.

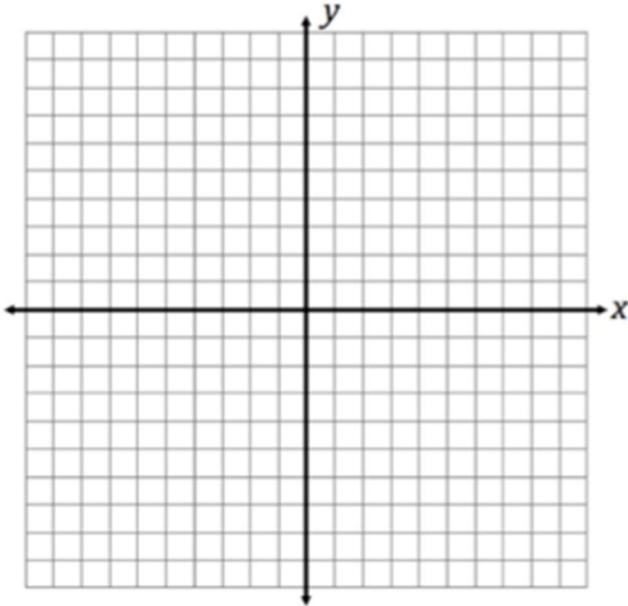
## Homework

1. Given the line  $2x + 3y = 12$  and the point  $(-5, 1)$ , find the following:
  - a. The equation of the line passing through the given point and parallel to the given line.
  
  
  
  
  
  
  
  
  
  
  - b. The equation of the line passing through the given point and perpendicular to the given line.
  
  
  
  
  
  
  
  
  
  
2. Given  $S(5, -4)$  and  $T(8, 12)$ :
  - a. Write an equation for the line through  $S$  and perpendicular to  $\overline{ST}$ .
  
  
  
  
  
  
  
  
  
  
  - b. Write an equation for the line through  $T$  and perpendicular to  $\overline{ST}$ .

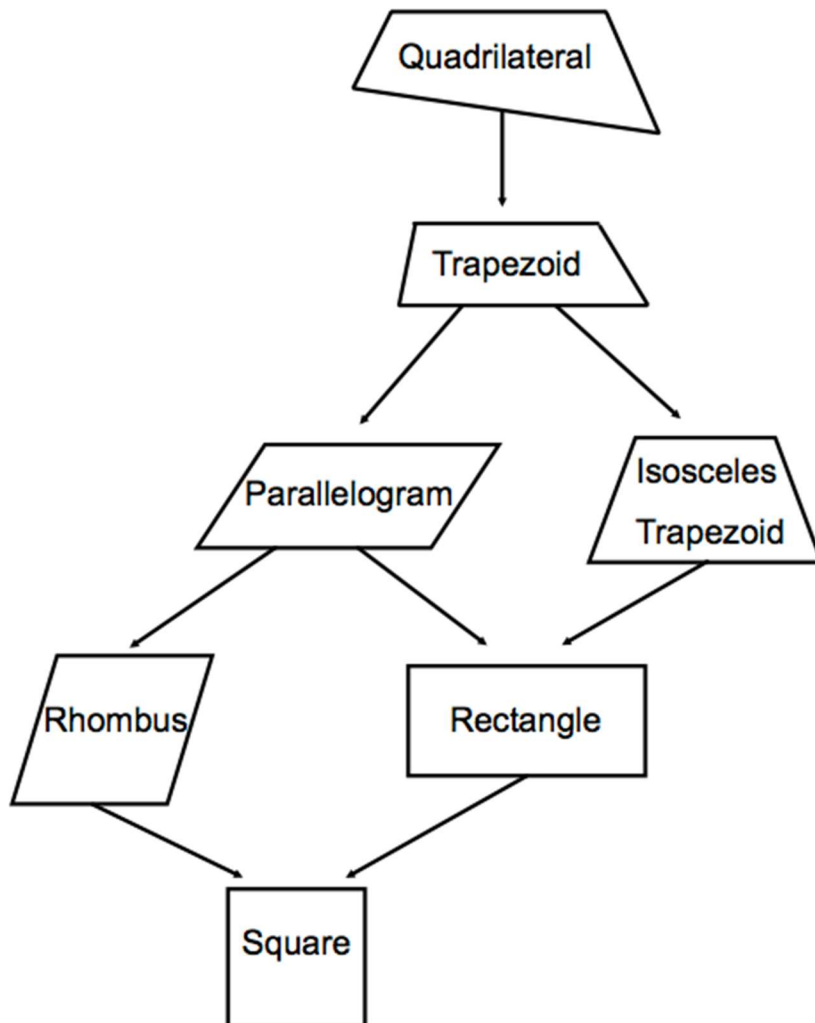
## Lesson 5: Coordinate Proofs Using Slope

### Opening Exercise

A triangle in the coordinate plane has vertices  $A(0, 10)$ ,  $B(-8, 8)$ , and  $C(-3, 5)$ . Is it a right triangle? If so, at which vertex is the right angle? (Use the axes below to help you!)



In Unit 3 we discussed the family of quadrilaterals and the properties they possess.

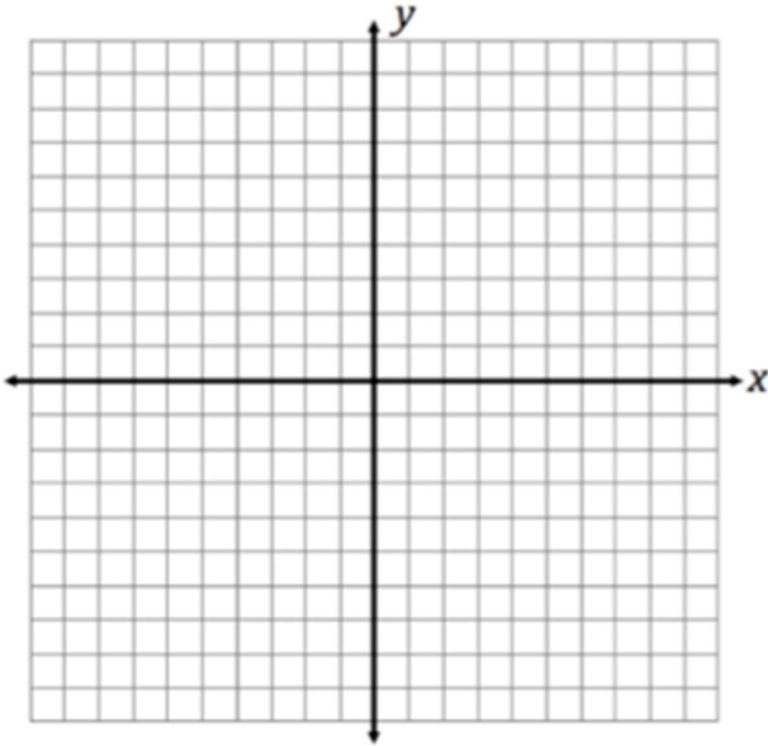


Now we can use what we understand about the relationship between the slopes of parallel and perpendicular lines to prove several types of these quadrilaterals!!

Slope Formula:  $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

### Example 1

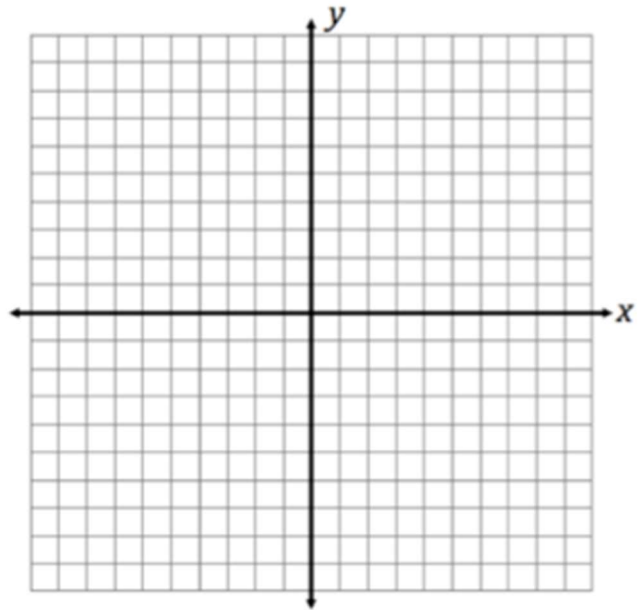
The vertices of quadrilateral  $BIRD$  are  $B(-1, -3)$ ,  $I(8, 0)$ ,  $R(3, 5)$  and  $D(0,4)$ . Using slope, show that  $BIRD$  is a trapezoid but *not* a parallelogram.



### Example 2

The points  $O(0,0)$ ,  $A(-4,1)$ ,  $B(-3,5)$ , and  $C(1,4)$  are the vertices of quadrilateral  $AOBC$ .

a. Prove  $AOBC$  is a parallelogram.

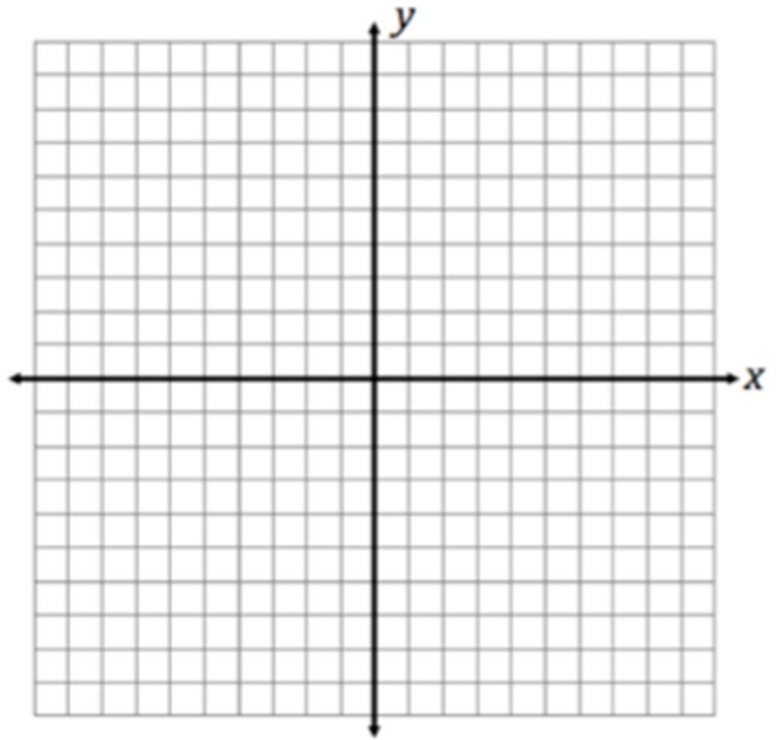


b. Is  $AOBC$  a rectangle? Justify.

## Homework

Quadrilateral  $ABCD$  has vertices  $A(-1, 5)$ ,  $B(5, 1)$ ,  $C(6, -2)$  and  $D(0, 2)$ .

- a.* Prove that  $ABCD$  is a parallelogram.



- b.* Is  $ABCD$  a square? Justify your answer