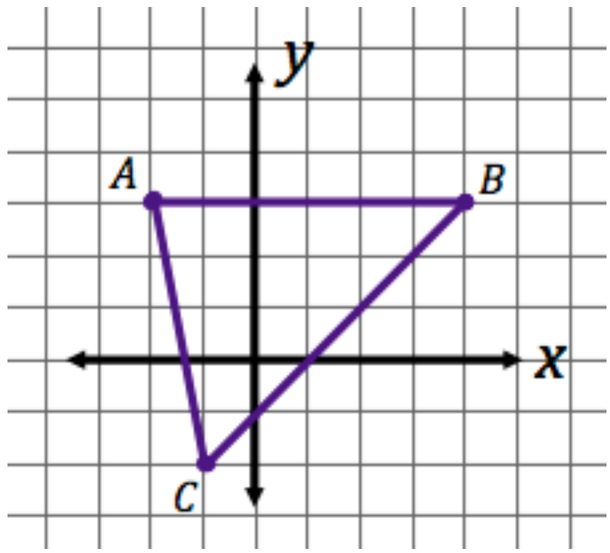
Unit 9
Applications of Distance

Lesson 1: Distance Formula

Opening Exercise

Given the triangle below with vertices $A(-2, 3)$, $B(4, 4)$ and $C(-1, -2)$.



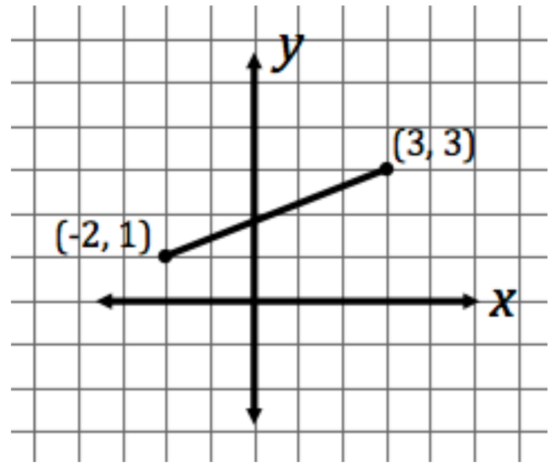
a. Calculate the exact perimeter of ΔABC.

b. Calculate the area of ΔABC.

Example 1

Given the segment pictured to the right, answer the following:

a. What is the change in *x*?



b. What is the change in *y*?

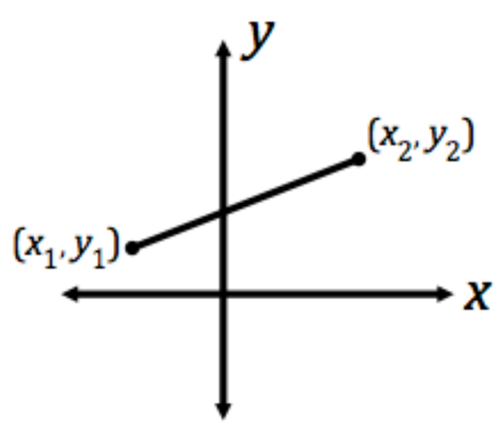
c. What is the slope?

d. Find the distance in simplest radical form.

Example 2

Given the segment pictured to the right, answer the following:

a. What is the change in *x*?



b. What is the change in *y*?

c. What is the slope?

d. What is the distance?

Distance Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Exercises

In 1-2, find the distance between the given coordinates using the distance formula. Leave answers in simplest radical form.

1. $A(2, 4)$ and $B(5, 7)$

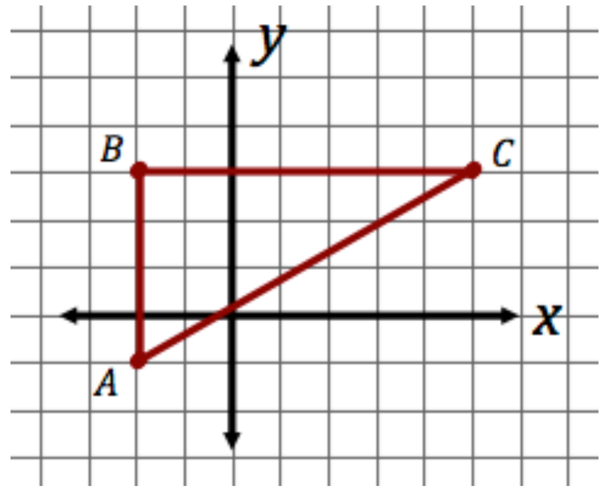
2. $C(-3, 8)$ and $D(-5, 1)$

3. Find the distance between points $(4a, 3b)$ and $(3a, 2b)$.

4. Find the radius of a circle whose diameter has endpoints (-3, -2) and (7, 8). Leave answer in *simplest radical form*.

Example 3

Given the pictured triangle, find:



a. Perimeter of the triangle in simplest radical form

b. Area of the triangle

Homework

In 1-2, find the distance between the given coordinates using the distance formula. Leave answers in simplest radical form.

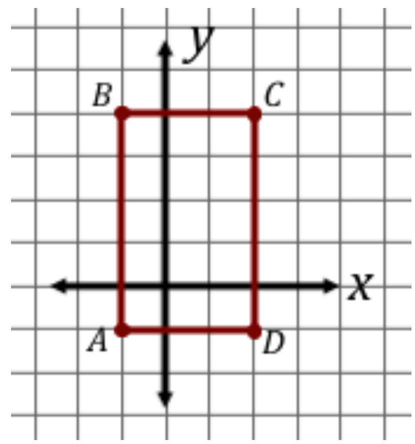
1. $J(4, -1)$ and $K(7, 5)$

2. $M(-4, 5)$ and $N(2, -5)$

3. Given the pictured polygon, find:

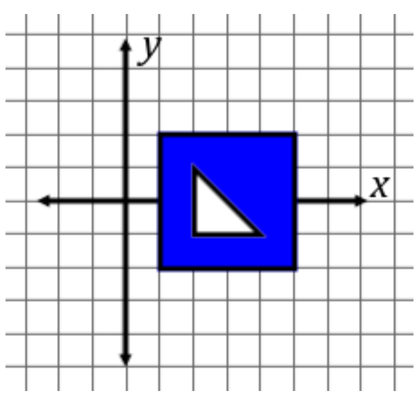
   a. Perimeter of the polygon

   b. Area of the polygon

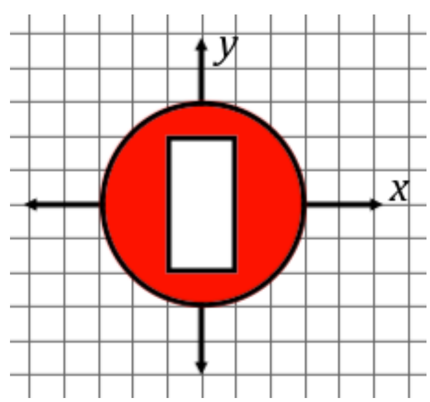


4. Find the exact areas of the shaded regions.

a.



b.

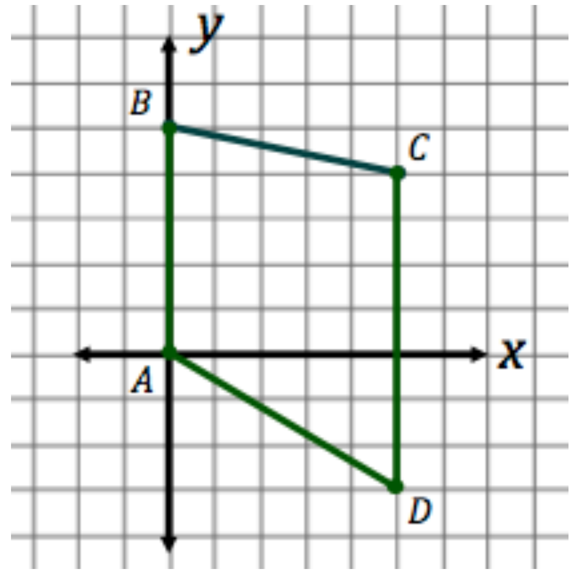


Lesson 2: Perimeter and Area of Polygons

Opening Exercise

Given the pictured quadrilateral, find the exact value of:

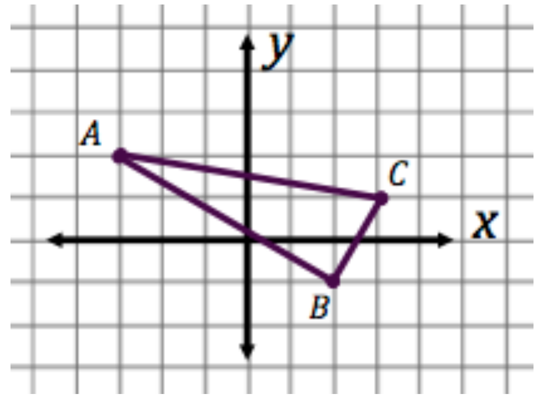
a. the perimeter of the quadrilateral



b. the area of the quadrilateral

How do we find the area of a triangle when the base and height are not easily identified?

One way to do this is to inscribe the triangle inside of a rectangle. This will create a shaded region problem using a rectangle with triangles that we can easily identify the bases and heights of.

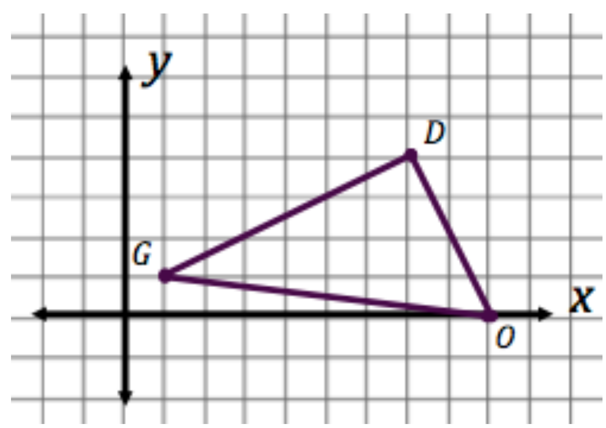


Example 1

Find the area of the triangle pictured above.

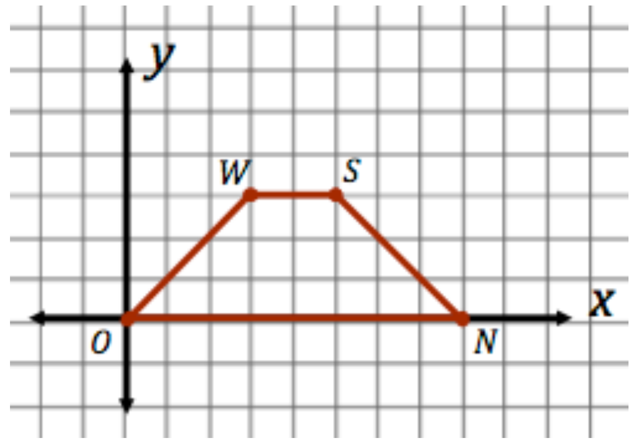
Example 2

Find the area of ΔDOG.



Exercise

Given trapezoid *SNOW*, find:



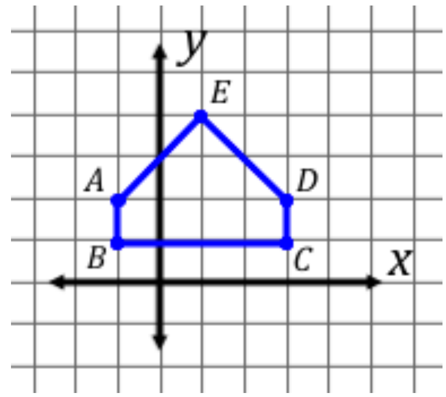
a. the perimeter of *SNOW.*

b. the area of *SNOW*

Homework

1. Given the pictured polygon, find the exact value of:

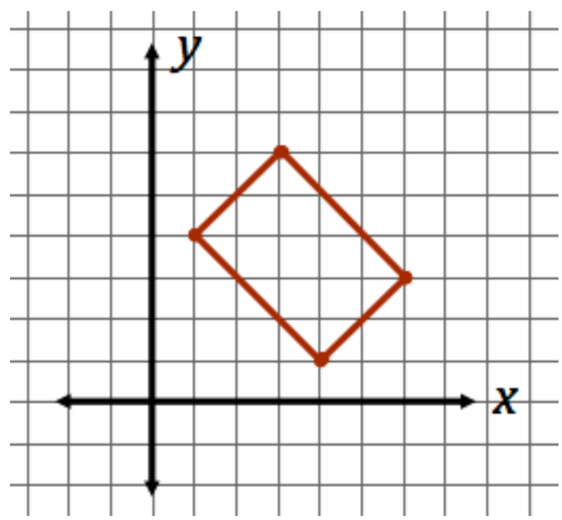
   a. Perimeter of the polygon



   b. Area of the polygon

2. Given rectangle *ABCD*:

   a. Identify the vertices.



   b. Find the exact value of the perimeter.

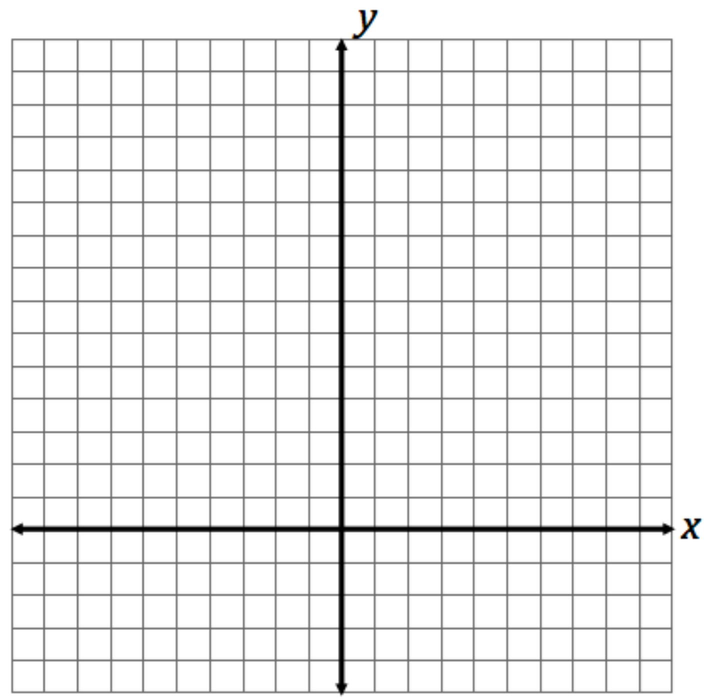
   c. Find the area.

Lesson 3: Perimeter and Area of Polygons Defined by Inequalities

Opening Exercise

Graph the system of inequalities on the grid provided:

$$y \le x + 6$$
$$y \le -2x + 12$$
$$y \ge 2x - 4$$
$$y \ge -x + 2$$



Find the vertices of the quadrilateral.

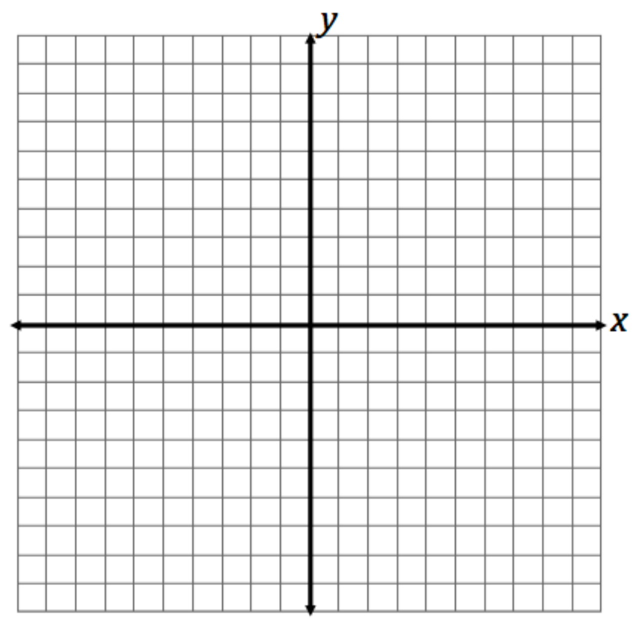
Find the perimeter of the quadrilateral to the *nearest hundredth*.

Find the area of the quadrilateral region.

Homework

Graph the system of inequalities on the grid provided:

$$y \le 5$$
$$y \ge -3$$
$$y \le 2x + 1$$
$$y \ge 2x - 7$$



Find the vertices of the quadrilateral.

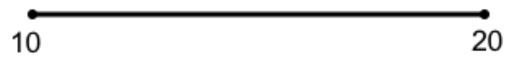
Find the area of the quadrilateral region.

Lesson 4: Dividing Segments Proportionately

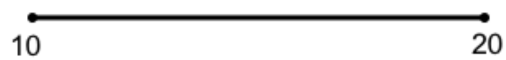
Opening Exercise

Given the pictured line segment, locate the point that is:

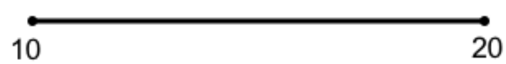
a. $\frac{1}{2}$ of the way from 10 to 20



b. $\frac{3}{4}$ of the way from 10 to 20



c. $\frac{2}{5}$ of the way from 10 to 20



Example 1

Using the problems above, show the work that is needed to answer each question:

How can we determine the number of units that are in each partition?

How can we find the distance from the starting point to the new point?

How can we refine this formula further to allow us to find the actual location of our answer?

Example 2

Using your knowledge from the previous example, locate the point that is $\frac{2}{3}$ of the way from 201 to 312.

Example 3

How can we find the point that is $\frac{a}{b}$ of the way from x_1 to x_2?

Example 4

Extending to 2-dimensions:

Given points *A*(-4, 5) and *B*(12, 13), find the coordinates of the point *D* that sits two-fifths of the way along $\overline{AB}$, closer to *B* than it is to *A*.

Exercises

1. Find the midpoint of $\overline{ST}$ given $S(-2, 8)$ and $T(10, -4)$.

2. Find the point on the directed segment from (-2, 0) to (5, 8) that divides it in the ratio of 1:3.

3. Given points $A(3, -5)$ and $B(19, -1)$, find the coordinates of point C such that $\frac{CB}{AC} = \frac{3}{7}$.

4. A robot is at position $A(40, 50)$ and is heading toward the point $B(2000, 2000)$ along a straight line at a constant speed. The robot will reach point B in 10 hours.

   a. What is the location of the robot at the end of the third hour?

   b. If the robot keeps moving along the straight path at the same constant speed as it passes through point B, what will be its location at the twelfth hour?

Homework

1. Find the midpoint of $\overline{MN}$ given $M(3, -1)$ and $N(11, -9)$

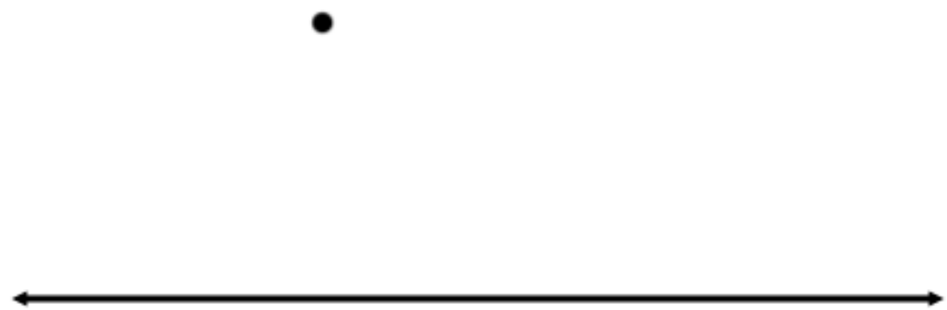
2. Given points $A(3, -5)$ and $B(19, -1)$, find the coordinates of point C that sits $\frac{3}{8}$ of the way along $\overline{AB}$, closer to A than to B.

3. What are the coordinates of the point on the directed line segment from $K(-5,-4)$ to $L(5,1)$ that partitions the segment into a ratio of 3 to 2?

Lesson 5: Perpendicular Bisectors & Distance From a Point to a Line

Opening Exercise

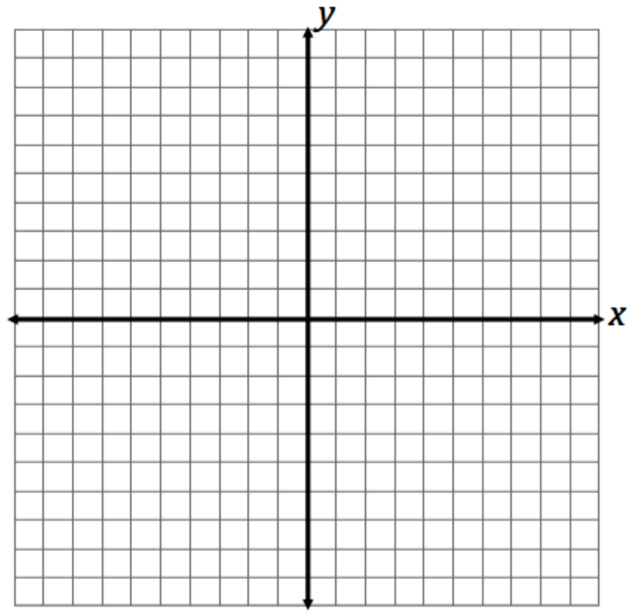
Construct the segment that is perpendicular to the given line using the given point as one of the endpoints.



How do you know that the segment you created is the shortest segment from the point to the line?

Example 1

Write an equation of the line that is the perpendicular bisector of the line segment having endpoints (3, -1) and (3, 5).



Step 1: Find the midpoint

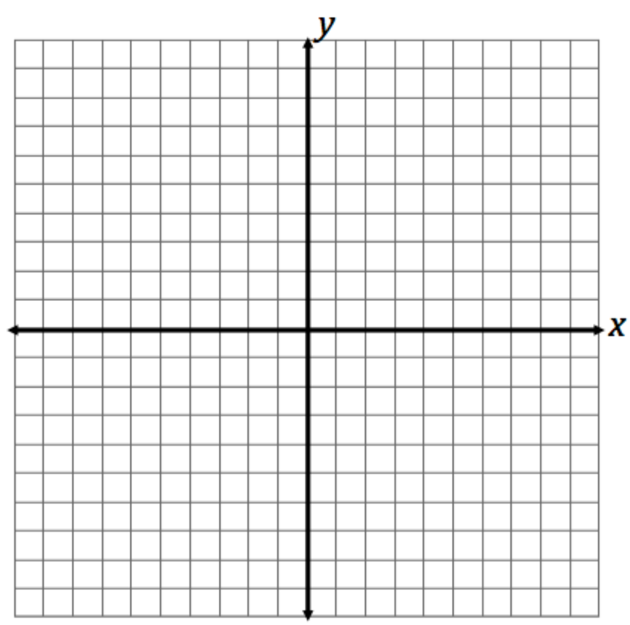
Step 2: Identify the slope

Step 3: Identify the perpendicular slope

Step 4: Write the equation of the line

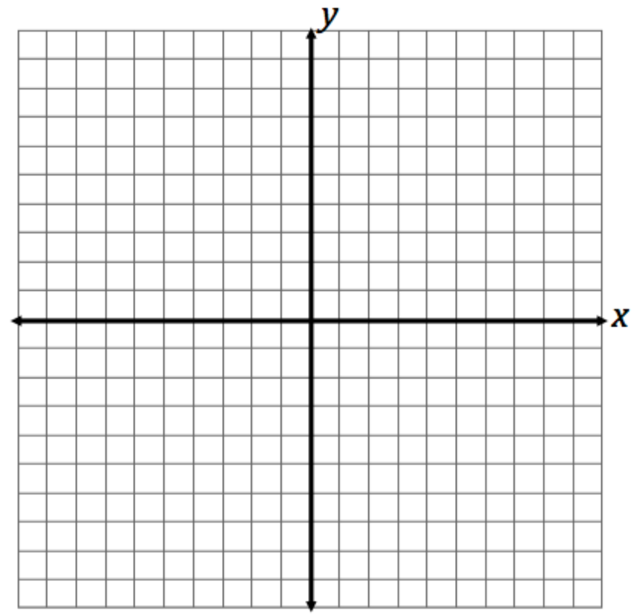
Example 2

Write an equation of the line that is the perpendicular bisector of the line segment having endpoints (-2, 7) and (8, -1).



Example 3

Find the exact distance between the point $P(0, 0)$ and the line $y = -x + 4$.



Step 1: Identify the slope

Step 2: Identify the perpendicular slope

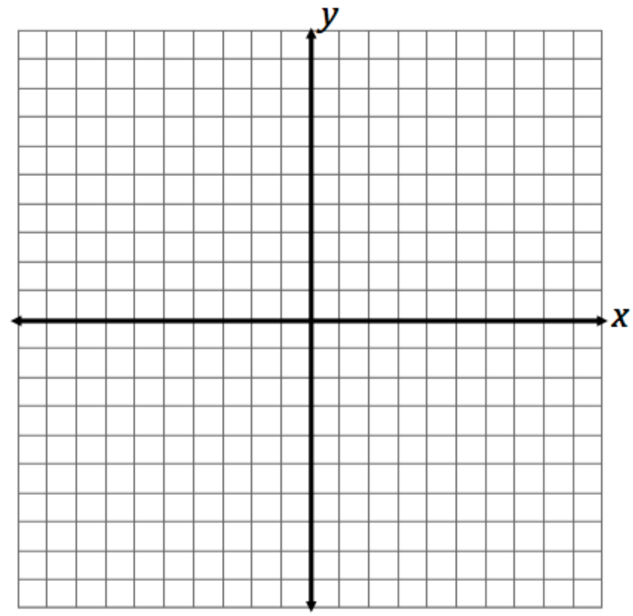
Step 3: Write the equation of the perpendicular line

Step 4: Find the intersection of the two lines

Step 5: Find the distance between the given point and the intersection point

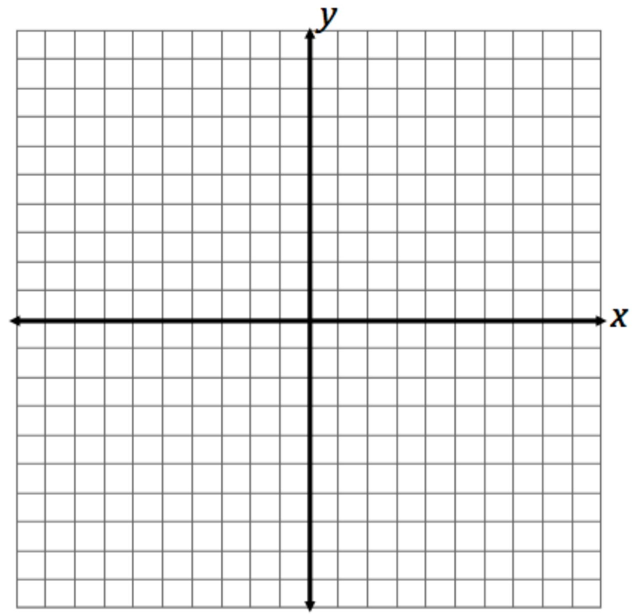
Example 4

Find the exact distance between the point $P(-2, 1)$ and the line $y = 2x$.

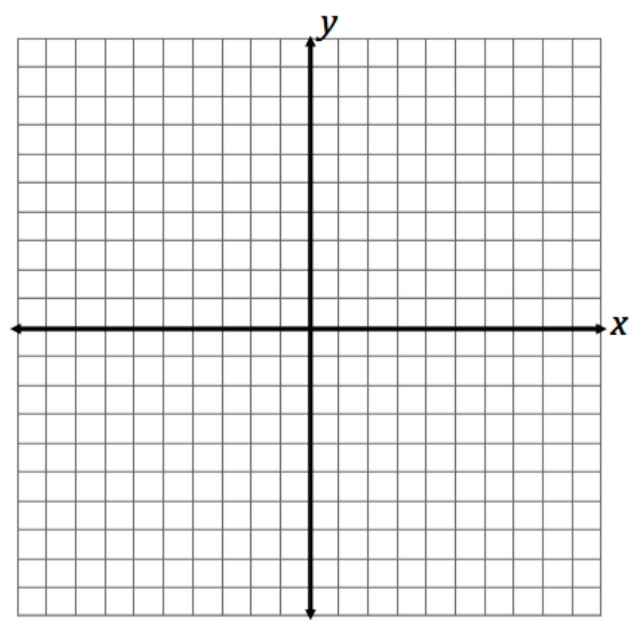


Homework

1. Write an equation of the line that is the perpendicular bisector of the line segment having endpoints (-6, 1) and (2, 5).



2. Find the exact distance between the point $P(0, 0)$ and the line $y = x + 10$.

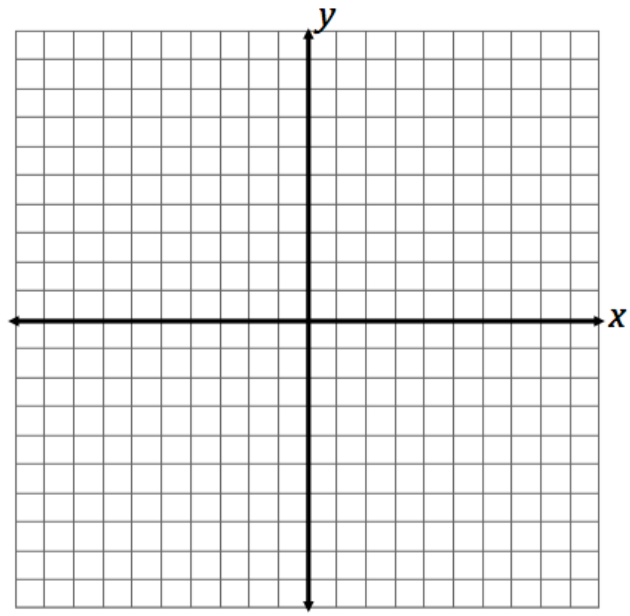


Lesson 6: Quadrilaterals and Trapezoids

Opening Exercise

The vertices of ΔWIN are $W(2, 1)$, $I(4, 7)$, and $N(8, 3)$. Using coordinate geometry, show:

a. ΔWIN is an isosceles triangle.

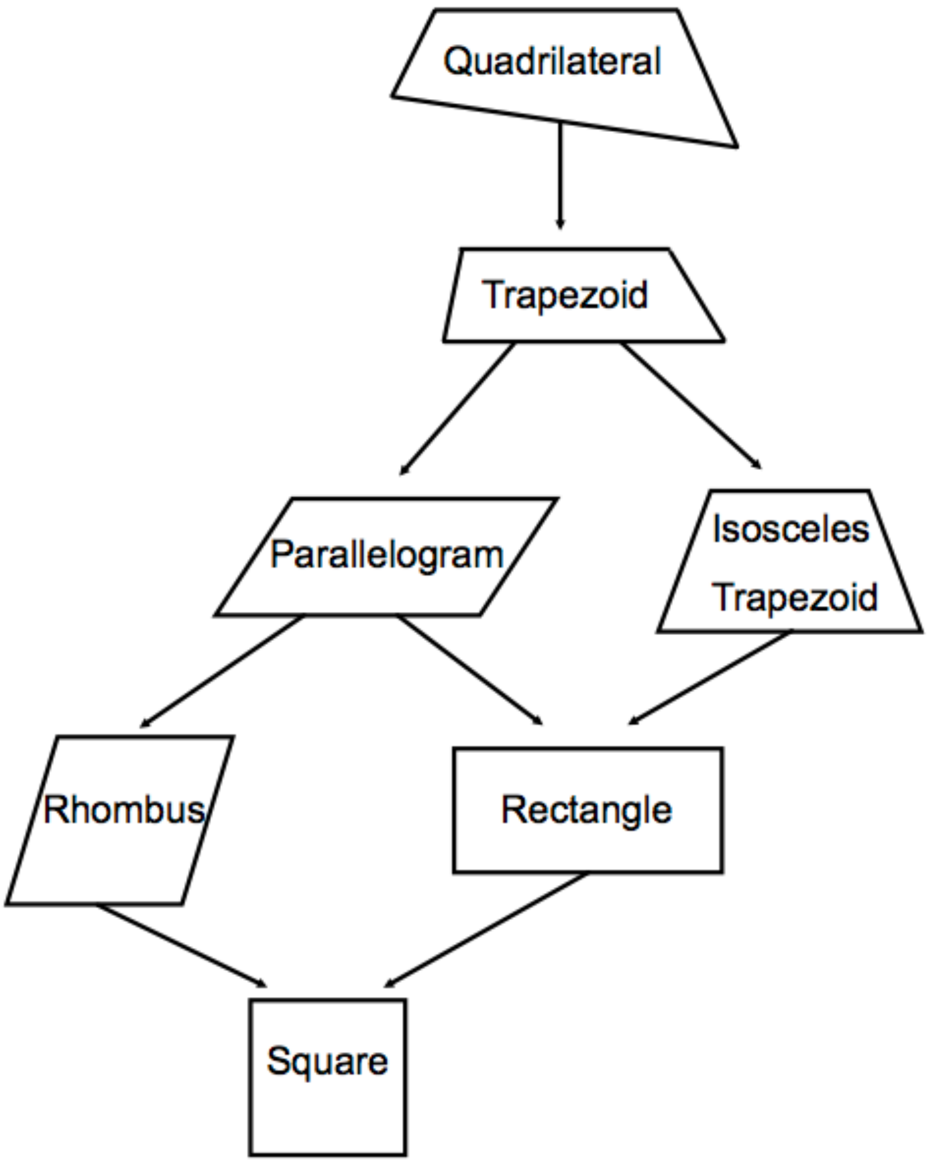


b. ΔWIN is *not* equilateral.

Example 1

Using the family of quadrilaterals pictured below, we are going to identify the properties of quadrilaterals, trapezoids and isosceles trapezoids.

Note that each figure has the properties of the figure(s) listed above it.

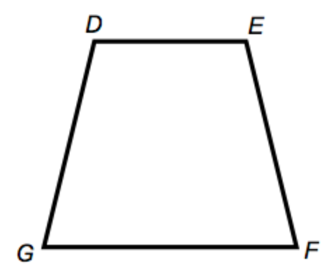


Exercises

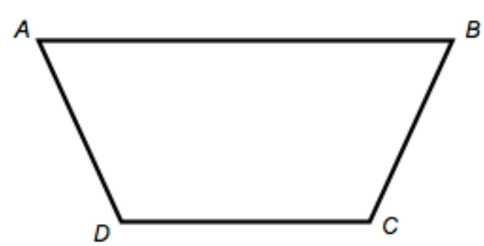
1. The measures of the angles of a quadrilateral are in the ratio 2:3:6:7. Find the number of degrees in the largest angle of the quadrilateral.

2. In isosceles trapezoid *ABCD*, *AD* = *BC*. What is $\angle A + \angle C$?

3. In the diagram of isosceles trapezoid *DEFG*, $\overline{DE} \parallel \overline{GF}$, $DE = 4x - 2$, $EF = 3x + 2$, $FG = 5x - 3$, and $GD = 2x + 5$. Find the value of x.



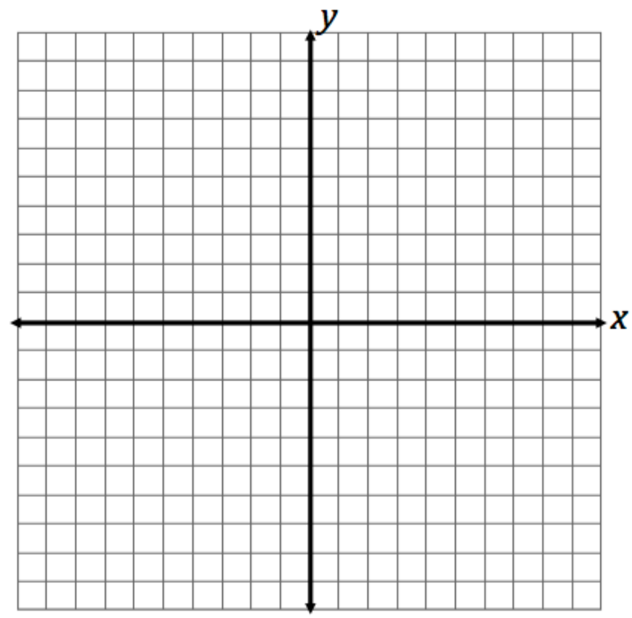
4. In the diagram of isosceles trapezoid *ABCD* with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. If $m\angle BAD = 2x$ and $m\angle BCD = 3x + 5$, find $m\angle BAD$.



Example

Give quadrilateral *ABCD* with vertices $A(4, 7)$, $B(8, 3)$, $C(6, -1)$, and $D(0, 5)$. Prove:

a. *ABCD* is a trapezoid

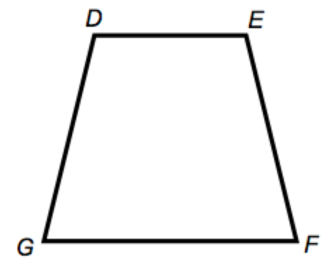


b. *ABCD* is an isosceles trapezoid

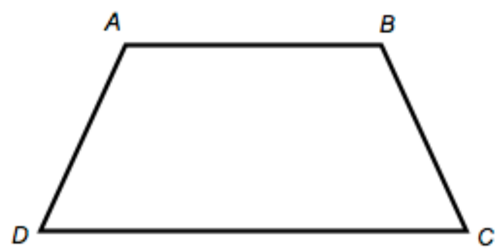
Homework

1. In quadrilateral *ABCD*, $m\angle A = 80$, $m\angle B = 2x$, $m\angle C = x$ and $m\angle D = 4x$. Find *x*.

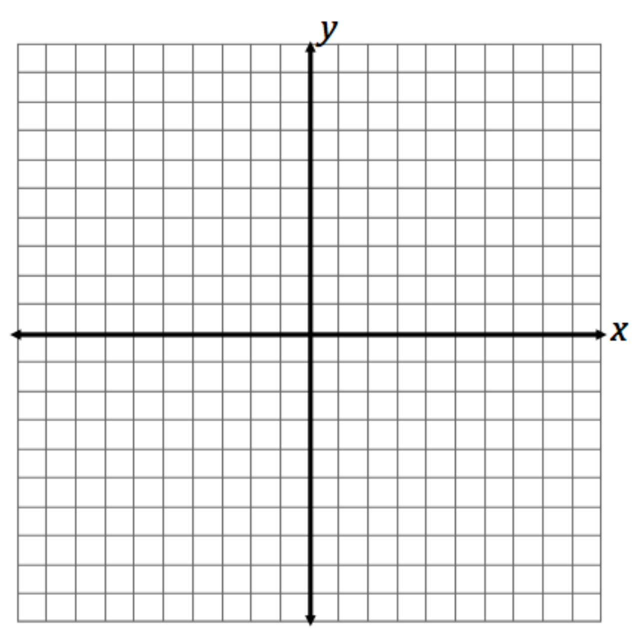
2. In the diagram of isosceles trapezoid *DEFG*, $\overline{DE} \parallel \overline{GF}$, $EF = 7x - 12$, $DE = x + 4$ and $GD = 2x + 3$. Find *DE*.



3. In the diagram of isosceles trapezoid *ABCD* with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. If diagonal $AC = 22$ and diagonal $BD = 6x + 4$. Find *x*.



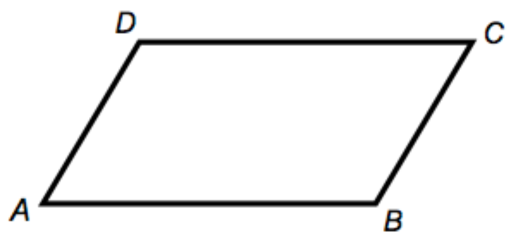
4. The vertices of quadrilateral *BIRD* are *B*(-1, -3), *I*(8, 0), *R*(3, 5) and *D*(0,4). Prove that quadrilateral *BIRD* is an isosceles trapezoid.



Lesson 7: Parallelograms and Rhombi

Opening Exercise

In Unit 3 we defined a parallelogram to be a quadrilateral in which both pairs of opposite sides are parallel. Using your knowledge of parallel lines, answer the following questions about parallelogram *ABCD:*



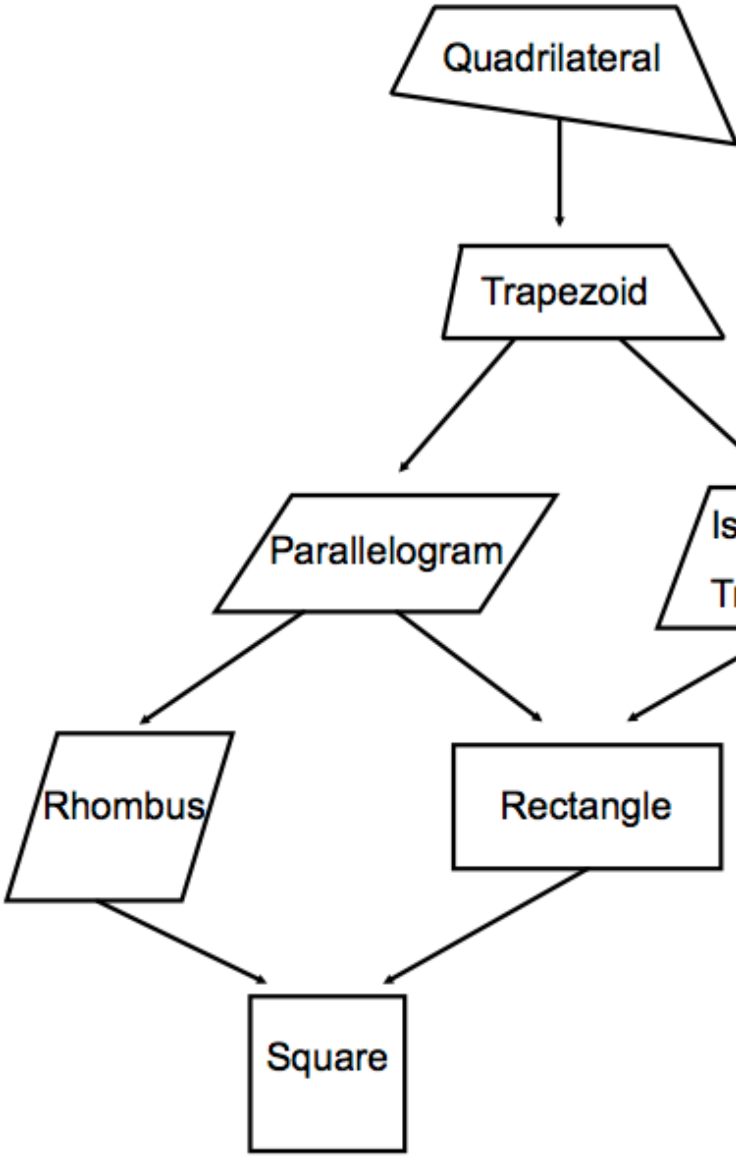
1. If $m\angle A = 40°$, find the measure of the remaining angles.

2. What do you notice about the consecutive angles of the parallelogram? Will this be true of all parallelograms? Explain.

3. What do you notice about the opposite angles of the parallelogram? Will this be true of all parallelograms? Explain.

4. What is the sum of the 4 angles of parallelogram *ABCD*? Will this be true of all parallelograms? Explain.

Using the family of quadrilaterals shown below, we are now going to fill in the properties of parallelograms and rhombi.



- 4 sides
- Angles add up to 360

- At least one pair of opposite sides are parallel
- Same side consecutive angles are supplementary

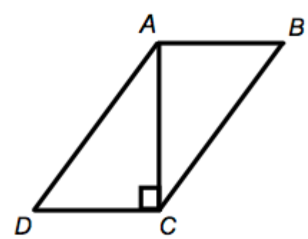
- At least one pair of opposite sides are ≅
- Diagonals are ≅
- Base angles are ≅

Exercises

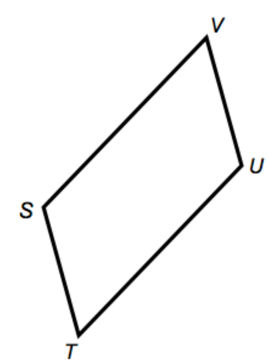
1. The measures of two consecutive angles of a parallelogram are in the ratio 3:7. Find the measure of an acute angle of the parallelogram.

2. In parallelogram *ABCD*, $\angle A = 2x + 50$ and $\angle C = 3x + 40$. Find the measure of $\angle A$.

3. In parallelogram *ABCD*, diagonal $\overline{AC} \perp \overline{CD}$. If $\angle ACB = 40$, find $m\angle ADC$.



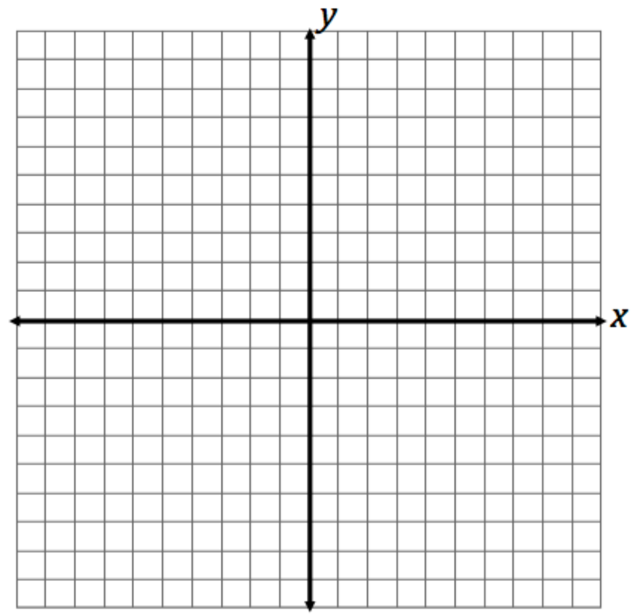
4. In the diagram of parallelogram *STUV*, $SV = x + 3$, $VU = 2x - 1$, and $TU = 4x - 3$. What is the length of $\overline{SV}$?



5. Find the length of the side of a rhombus whose diagonals measure 12 and 16.

Example

Given: ΔABC with vertices $A(-6, -2)$, $B(2, 8)$, and $C(6, -2)$, $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.



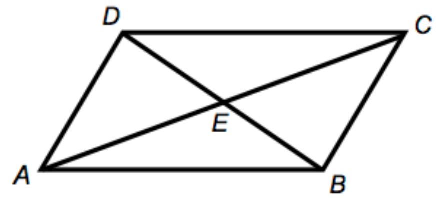
Prove: $ADEF$ is a parallelogram

$ADEF$ is not a rhombus

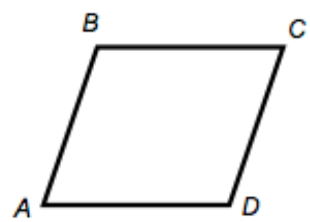
Homework

1. In parallelogram *ABCD*, $\angle A = 2x - 10$ and $\angle B = 5x + 15$. Find *x.*

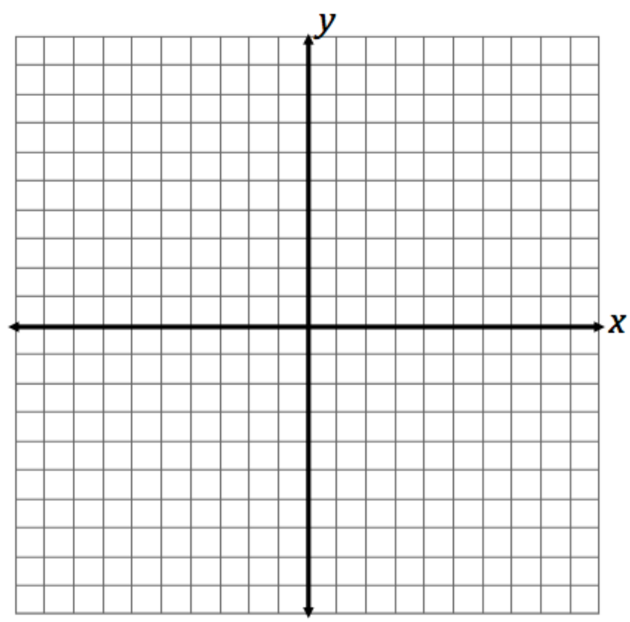
2. In the accompanying diagram of parallelogram *ABCD*, diagonals $\overline{AC}$ and $\overline{DB}$ intersect at *E*, *AE* = 3*x* – 4 and *EC* = *x* + 12. What is the value of *x*?



3. In the accompanying diagram of rhombus *ABCD*, the lengths of the sides *AB* and *BC* are represented by 3*x* – 4 and 2*x* + 1, respectively. Find the value of *x*.



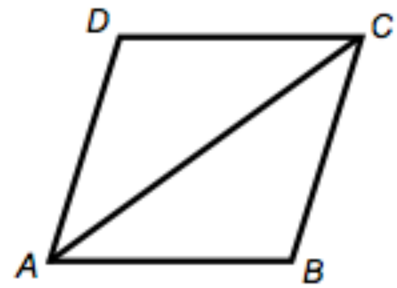
4. The vertices of quadrilateral *MATH* are *M*(-2,3), *A*(-1,7), *T*(3,8) *H*(2,4). Prove that *MATH* is a rhombus.



Lesson 8: Rectangles and Squares

Opening Exercise

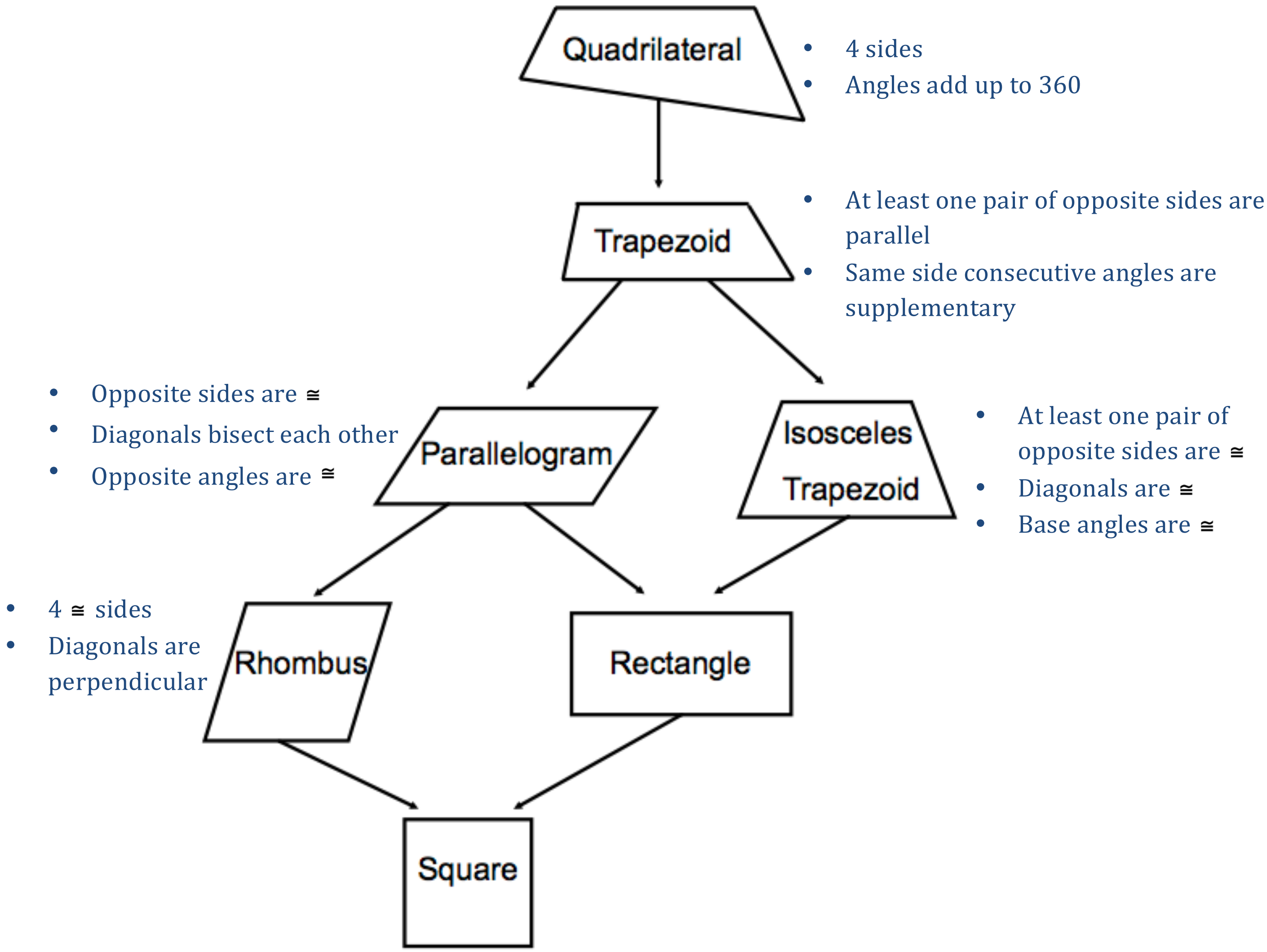
In the accompanying diagram of rhombus *ABCD*, diagonal $\overline{AC}$ is drawn. If $m\angle CAB = 35$, $AD = 5x + 7$ and $CD = 6x - 1$, find:



a. $m\angle ADC$

b. $\overline{AD}$

Using the family of quadrilaterals shown below, we are now going to fill in the remaining properties:

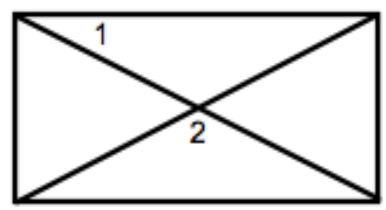


Exercises

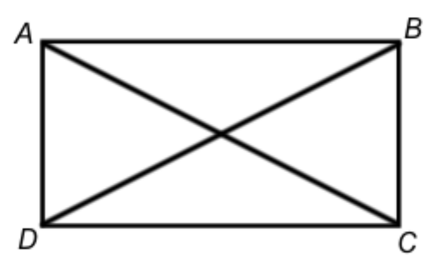
1. The lengths of the diagonals of a rectangles are represented by $2x + 3$ and $4x - 11$. Find the value of x.

2. The diagonals of rectangle *ABCD* intersect at *E*. If $DE = 3x + 1$ and $AC = 5x + 4$, find the value of x.

3. As shown in the accompanying diagram, a rectangular gate has two diagonal supports. If $m\angle 1 = 42$, what is $m\angle 2$?

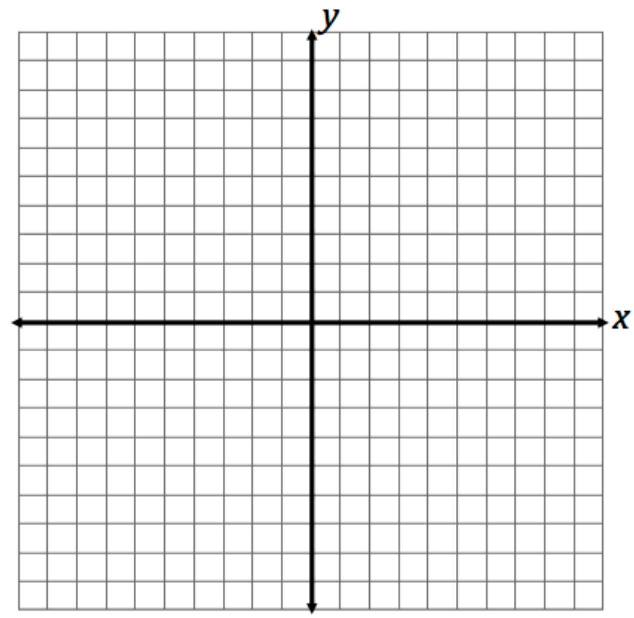


4. In the accompanying diagram of rectangle *ABCD*, $m\angle BAC = 3x + 4$ and $m\angle ACD = x + 28$. What is $m\angle CAD$?



Example

If the coordinates of quadrilateral *ABCD* are *A*(-1,0), *B*(3,3), *C*(6,-1), and *D*(2,-4), prove that *ABCD* is a square.



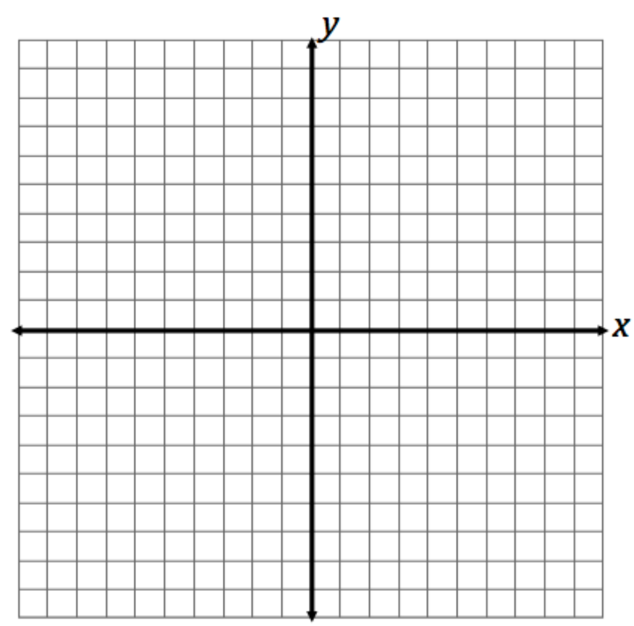
Homework

1. In rectangle *ABCD*, diagonal $AC = x + 10$ and diagonal $BD = 2x - 30$. Find *x.*

2. A rectangular lot that is 60 feet by 80 feet has a straight diagonal pathway. What is the length, in feet, of the diagonal pathway?

3. The coordinates of quadrilateral *CAKE* are *C*(-3,2), *A*(-1,5), *K*(5,1), and *E*(3,-2).

   a. Show that *CAKE* is a rectangle.



   b. Is *CAKE* a square? Justify your answer.