Chapter 6 – Differential Equations and Mathematical Modeling

6.1 Antiderivatives and Slope Fields

Def: An equation of the form:

$$\frac{dy}{dx} = y \ln x$$

which contains a derivative is called a ***Differential Equation***.

- In this equation, you are to find a function y in terms of x (i.e. $y = f(x)$)
- When we are given the derivative of a function and its value at a given point:

  Ex. $\frac{dy}{dx} = 2x - 2 \quad f(4) = 10$

  This is called an ***initial value problem***.

  - The value of f for one value of x is the initial condition of the problem.
- Finding ALL functions y that satisfy the differential equation is called **solving the differential equation.**

Ex. Suppose \$100 is invested in an account that pays 5.6% interest compounded continuously. Find a formula for the amount in the account at any time t.

Because the interest is compounded on the original value, the equation would be:

$$\frac{dy}{dx} = 0.056y \quad \text{with } y(0) = 100$$

How do you solve it?

- Well one way is to think of a function whose derivative is a multiple of itself.
- Another is to get the multiple of itself.
  - This would be the exponential function:

    $$y(t) = Ce^{0.056x}$$

    Proof:

    $$y = Ce^{0.056x}$$

    $$\frac{dy}{dx} = C(0.056e^{0.056x}) = 0.056(Ce^{0.056x}) = 0.056y(t)$$
  - Using the pre-existing condition:

    $$y(t) = Ce^{0.056x}$$

    $$y(0) = Ce^{0.056(0)}$$

    $$100 = Ce^{0}$$

    $$100 = C$$
  - This leads to the equation: $y = 100e^{0.056x}$

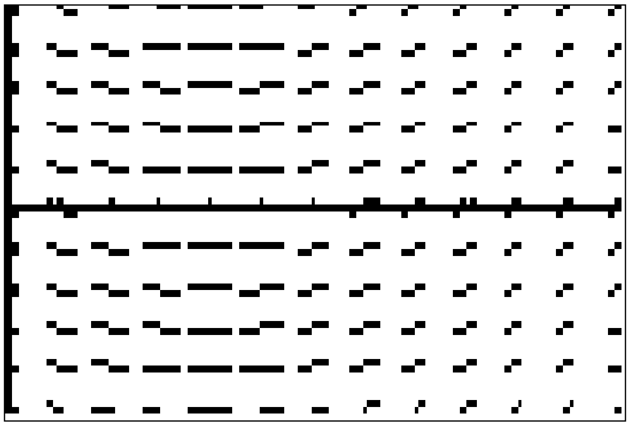
- The initial solution $y(t) = Ce^{0.056t}$ is called the family of solutions. Depending on the value of C.
  - Because of the pre-existing condition $y(0) = 100$, we could find the value of C.
- Is there a way to see the family of solutions for a particular integral?? Yes, it is called Slope Fields.

Def: **Slope Field**

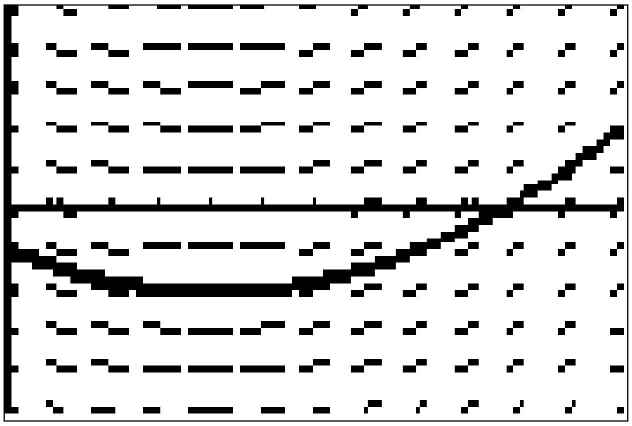
A slope field (or directional field) for a differential equation

$$\frac{dy}{dx} = f(x, y)$$

is a plot of short line segments with slopes $f(x, y)$ for a lattice (set) of points (x, y) in the plane.

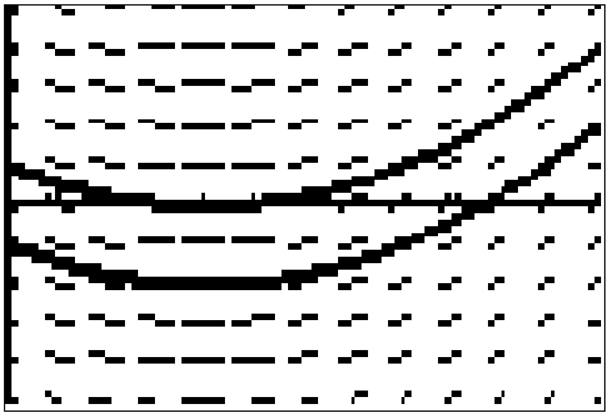


Slope field for $\frac{dy}{dx} = 2x - 2$



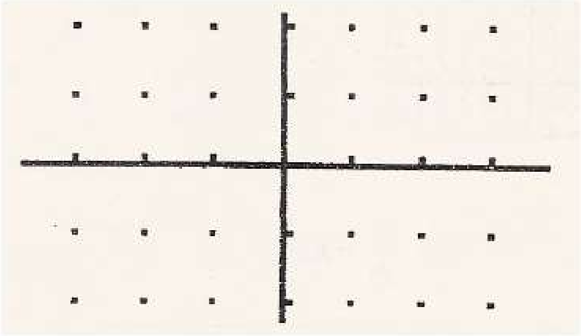
Same slope field with $y = x^2 - 2x - 1$

- Each of the little lines in the slope field represent a tangent line to one of the family of curves for the differential equation.
- Here is the same slope field with two curves: $y = x^2 - 2x - 1$ and $y = x^2 - 2x + 1$

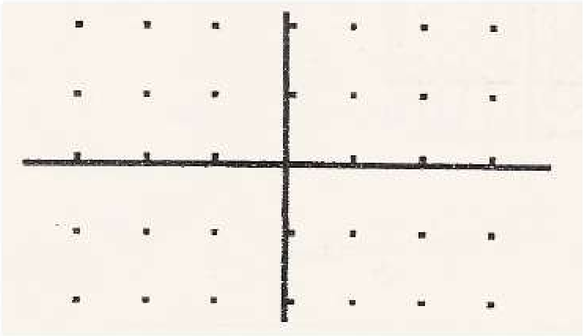


Draw a slope field for each of the following differential equations. Each tick mark is one unit.

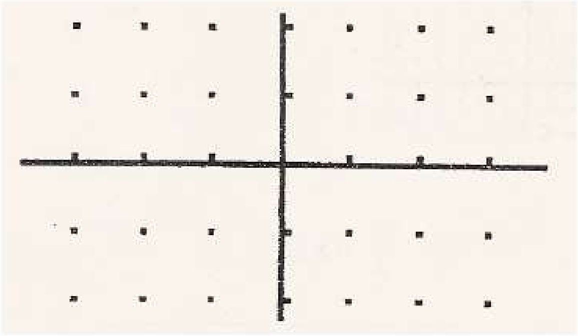
1. $\frac{dy}{dx} = x + 1$



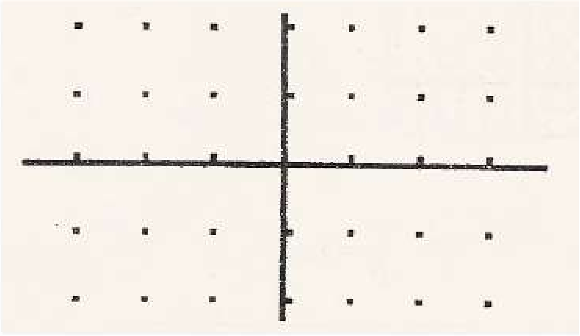
2. $\frac{dy}{dx} = 2y$



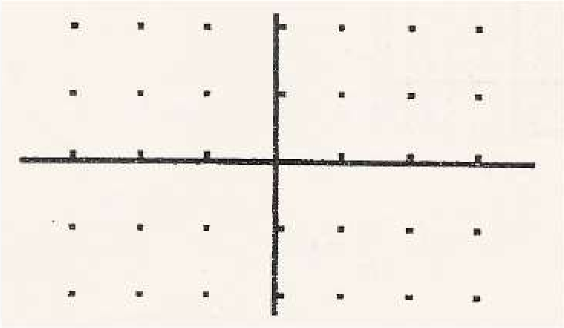
3. $\frac{dy}{dx} = x + y$



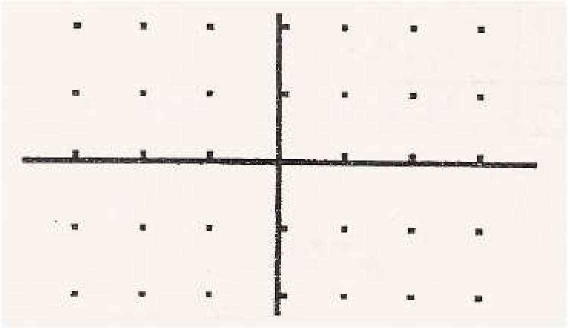
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y - 1$

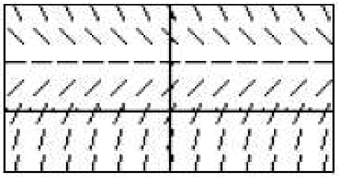


6. $\frac{dy}{dx} = -\frac{y}{x}$

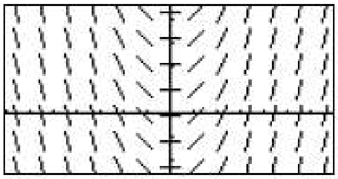


Match the slope fields with their differential equations.

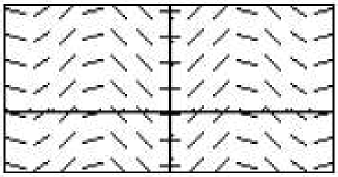
(A)



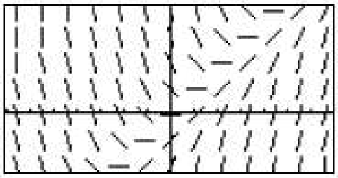
(B)



(C)



(D)



7. $\frac{dy}{dx} = \sin x$

8. $\frac{dy}{dx} = x - y$

9. $\frac{dy}{dx} = 2 - y$

10. $\frac{dy}{dx} = x$

<u>Homework</u>: Pg. 327-329 #1-3, 5, 7-10, 12-24(3's), 25-39odd, 51, 56

6.2 Antidifferentiation by Substitution

Def: ***Indefinite Integral***

The set of all antiderivatives of a function $f(x)$ is the ***indefinite integral*** of f with respect to x and is denoted by:

$$\int f(x)dx$$

And if $F(x)$ is an antiderivative of $f(x)$ as defined in the Fundamental Theorem, then

$$\int f(x)dx = F(x) + C$$

where $F'(x) = f(x)$ and C is ANY constant value.

- C is called the ***constant of integration*** and is an arbitrary constant

Ex. Evaluate:

1. $\int 2x dx$

2. $\int \cos x dx$

3. $\int (x^2 + 4x - 5) dx$

4. $\int e^x dx$

5. $\int \frac{1}{\sqrt{x}} dx$

- Below is a table of some basic Indefinite Integrals:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int e^{kx} dx = \frac{e^{kx}}{k} + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$	$\int \cos kx dx = -\frac{\sin kx}{k} + C$	$\int \sec^2 kx dx = \frac{\tan kx}{k} + C$
$\int \csc^2 kx dx = -\frac{\cot kx}{k} + C$		

ex. Evaluate: $\int e^{-4x} dx$ $\int \cos \tfrac{1}{2} x dx$

Properties of Indefinite Integrals:

Let k be a real number:

1. *Constant Multiple Rule:* $\int kf(x)dx = k\int f(x)dx$

   If $k=-1$, *then,* $\int -f(x)dx = -\int f(x)dx$

2. *Sum and Difference Rule:* $\int (f(x)\pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

Ex. Evaluate $\int (3x^2-4x+5)dx$

ex. A right cylindrical tank with radius 5 ft and height 16 ft was initially filled with water is being drained at a rate of $0.5\sqrt{x}$ ft^3/min, where x is the depth of the water. Find a formula for the depth and the amount of water in the tank at any time t. How long will it take the tank to empty?

We can now solve this differential equation:

Ex. A heavy projectile is fired straight up into the air from a platform 3 meters above the ground with an initial velocity of 160 *m*/sec. Assume that only the force affecting the projectile during its flight is gravity, which produces a downward acceleration of 9.8 *m*/sec^2. If $t = 0$ when the projectile is fired, find a formula for the projectile's

(a) velocity as a function of time *t*
(b) height above the ground as a function of time *t*.

The acceleration function is a constant function defined above:

$a(t) = -9.8$ negative since it is a downward acceleration

But, acceleration is the derivative of the velocity:

$$a(t) = v'(t)$$
$$\therefore$$
$$v(t) = \int a(t)dt$$
$$v(t) = \int -9.8dt = -9.8t + C$$

Using the information above (initial velocity is 160):

$$v(0) = -9.8(0) + C$$
$$160 = C$$

We get the velocity function to be: $v(t) = -9.8t + 160$

The velocity function is the derivative of the position function (the height of the projectile).

$$v(t) = s'(t)$$
$$\therefore$$
$$s(t) = \int v(t)dt$$
$$= \int (-9.8t + 160)dt$$

<u>Aside</u>:

$$\int -9.8tdt = -9.8\int tdt$$
$$= -9.8 \cdot \frac{t^2}{2}$$
$$= \frac{-9.8t^2}{2}$$
$$= -4.9t^2 + C$$

$$s(t) = -4.9t^2 + 160t + C$$

Using the information above,

$$s(0) = -4.9(0^2) + 1690(0) + C$$
$$3 = C$$

We get the position function to be: $s(t) = -4.9t^2 + 160t + 3$

Homework: Pg 337-338 QR #1-10; Ex #1-10, 17-24

6.2b Integration by Substitution (Day 2)

- When *u* is a differentiable function of *x* and *n* is a real number not equal to –1, then the Chain Rule gives us:

$$\frac{d}{dx}\left(\frac{u^{n+1}}{n+1}\right)=u^{n}\frac{du}{dx}$$

- Switch the direction, we get:

$$\int\left(u^{n}\frac{du}{dx}\right)dx=\frac{u^{n+1}}{n+1}+C \quad \text{for } n\neq -1$$

- A change in variable can often make an unfamiliar or difficult looking integral into one that we can evaluate.
    - This method is called ***substitution method of integration***.

Ex. Evaluate $\int (x+2)^{5}\,dx$

Ex. Evaluate $\int \sqrt{4x-1}\,dx$

Ex. Evaluate: $\int \cos(3x-2)dx$

Ex. Evaluate $\int \frac{1}{\cos^2 2x}dx$

Ex. Evaluate $\int \tan x dx$

To evaluate integrals that are not "simple":

1. Let u = an expression whose derivative in also in the expression
2. Take the derivative of the equation written in step 1, and solve for du
3. It should be the other expression in the integrand and dx
4. Make you substitutions and integrate with respect to u (this should be a "simple" integral).
5. Substitute back into the u and add a "+C" if it is an indefinite integral.

Ex. Evaluate: $\int \sin^3 x \cos x dx$

$\int (x^2+2x-3)^2(x+1)dx$

- What if it a definite integral? Same process, just plug the value of u back in then use the bounds.

Ex. Evaluate $\int_{0}^{\pi/4} \tan x \sec^2 x \, dx$

Let $u = \tan x$

Ex. Evaluate $\int_{-1}^{1} 3x^2 \sqrt{x^3+1} dx$

- There are some people who think that when you make the u-substitution, that you change the bounds according the substitution.

$$\int_{a}^{b} f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

  - Personally, I don't like this method and I don't use it

Separable Differential Equations

- A differential equation $y' = f(x, y)$ is ***separable*** if f can be expressed as a product of a function of x and a function of y.
  - This means:

$$\frac{dy}{dx} = g(x)h(y)$$

- If $h(y) \neq 0$, then we can **separate the variables** by dividing both sides of the equation by $h(y)$:

$$\frac{1}{h(y)}\frac{dy}{dx} = g(x)$$

- Integrating both sides of the equation with respect to x, we get:

$$\int \frac{1}{h(y)}\frac{dy}{dx}dx = \int g(x)dx$$

$$\int \frac{1}{h(y)}dy = \int g(x)dx$$

- With the variables separated, we can integrate each side of the equation and write the function expressing y as a function of x.

Ex. Solve the differential equation

$$\frac{dy}{dx} = 2x(1+y^2)e^{x^2}$$

Ex. Solve the differential equation

$$\frac{dy}{dx} = x\sqrt{y}\cos^2\sqrt{y}$$

Homework: Day 1: Pg 337 – 338 #25– 29, 32, 33, 39, 41, 43, 47, 45-66(3s)
Day 2 Worksheet: Integration by Substitution

6.4 Exponential Growth and Decay

Def: ***Separable Differential Equations***

A differential equation of the form $\frac{dy}{dx} = f(y)g(x)$ is called a ***separable***. We separate the variables by writing the equation into the following form:

$$\frac{1}{f(y)}dy = g(x)dx$$

We then integrate both sides using Antidifferentiation with respect to the variable on each side.

Ex. Find y if $\frac{dy}{dx} = (xy)^2$

Ex. Find y if $\frac{dy}{dx} = x^2y$

The Law of Exponential Change

If y changes at a rate proportional to the amount present ($\frac{dy}{dx} = ky$) and $y = y_0$ when $t = 0$, then

$$y = y_0e^{kt}$$

If $k > 0$, then this is exponential growth
If $k < 0$, then this is exponential decay

k represents the RATE CONSTANT of the equation

Continuous Compound Interest

- The formula for compound interest on an account with A_0 at a rate of r% over a time period of t years in which there are k times it is compounded is:

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt}$$

- This formula is used when you have a set number of times when the interest is compounded, k.
  - The interest rate never changes so the interest is proportional on the amount that is in the account.
  - If we were to compound it continuously, then it would be considered an exponential growth. Therefore the formula would be:

$$A(t) = A_0 e^{kt}$$

Ex. Suppose you have $5000 is an IRA that earns 6.5% annually. How much would you have after 8 years if the interest was compounded

(a) quarterly (b) monthly

(c) daily (d) continuously

Radioactivity

- Radioactivity works the exact same way as interest, except it is a decay.

If you have an initial amount y_0 of a radioactive substance, then the amount still present at any later time t will be

$$y = y_0 e^{-kt}$$

- The **half-life** of a radioactive substance is the amount of time for half of the radioactive nuclei present in a sample it decay.

Ex. Find the half-life formula of any radioactive substance as a function of k

Homework: Pg. 357 #1-10, 13,14

6.4 Exponential Growth and Decay (con't)

Ex. Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear reactor accident in 1986. How long will it take for the 10 grams to decay to 1 gram if the half-life of Pu-239 is 24,360 yrs?

Ex. Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after day 4. Approximately how many flies were in the original population?

Ex. Scientists who do carbon-14 dating use 5700 yrs for its half-life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

$$\text{Half-life is } 5700 = \frac{\ln 2}{k}$$

Newton's Law of Cooling

If T is the temperature of an object at time t, and T_s, is the surrounding temperature, then

$$\frac{dT}{dt} = -k(T - T_s)$$

Since dT represents the change in temperature, then $dT = d(T - T_s)$, we can rewrite the above equation:

$$\begin{aligned} \frac{dT}{dt} &= -k(T - T_s) \\ \frac{d}{dt}(T - T_s) &= -k(T - T_s) \end{aligned}$$

This new equation is of the form $y' = -ky$ which is solved as follows:

$$\begin{aligned} y' &= ky \\ \frac{dy}{dt} &= ky \\ \frac{dy}{y} &= kdt \\ \ln|y| &= kt + C \\ e^{\ln|y|} &= e^{kt+C} \\ |y| &= e^C \cdot e^{kt} \\ y &= \pm Ce^{kt} \\ y &= Ce^{kt} \end{aligned}$$

Law of Exponential Change

Since C is a constant, we can remove the ±

Therefore:

$$\begin{aligned} \frac{d}{dt}(T - T_s) &= -k(T - T_s) \\ T - T_s &= (T_0 - T_s)e^{-kt} \end{aligned}$$

where T_0 is the original temperature at $t = 0$.

This is Newton's Law of Cooling

Ex. Let y represent the temperature (in °F) of an object in a room whose temperature is kept at a constant 60°. If the object cools from 100° to 90° in 10 minutes, how much longer will it take for its temperature to decrease to 80°

ex. A deep dish apple pie whose internal temperature is 220°F when removed from the oven, was set on a 40°F porch to cool. Fifteen minutes later, the pie's internal temperature was 180°F. How long did it take the pie to cool from there to 70°F?

Ex. A pan of warm water (46°C) is put in the refrigerator. 10 minutes later, the water's temperature was 39°C. 10 minutes later, it was 33°C. Estimate the temperature of the refrigerator.

Resistance Proportional to Velocity

- In Physics, the resistance encountered by a moving object (i.e. a car coasting to a stop) is proportional to the object's velocity.
    - The slower the object moves, the less resistance against the forward progress
- Mathematically:

Force = mass × acceleration

$$F = ma$$
$$F = mv'$$
$$F = m\frac{dv}{dt}$$

- Since the resistance force is proportion to the velocity, then

$$m\frac{dv}{dt} = -kv$$
$$\frac{dv}{dt} = -\frac{k}{m}v$$
$$\frac{dv}{v} = -\frac{k}{m}dt$$

which when solved (just like we did earlier), results in:

$$v = v_0 e^{-(k/m)t}$$

ex. For a 50-kg ice skater, $k = 2.5$ kg/sec. Answer the following:

(a) How long will it take the skater to coast from 7 m/sec to 1 m/sec?

(b) How far will the skater coast before coming to a stop?

Homework: Day 1: Pg 357-358 #11(a), 12, 18 – 20, 26 – 28

Day 2: Pg. 357-360 #14-17, 21-24, 29, 32, 35, 36, 39, 42, 45, 46

6.5 Logistic Growth

Partial Fractions

ex. Evaluate: $\int \frac{5x-3}{x^2-2x-3}dx$

- There is no *u*-substitution that can be made for this
- No trig substitution as well.
- To evaluat.e this integral, we need to use the ***method of partial fractions***.
    - We will rewrite this fraction as the sum/difference of simpler fractions.
    - Factor the denominator: $(x-3)(x+1)$
    - Now rewrite:
      $$\frac{5x-3}{x^2-2x-3}=\frac{A}{x-3}+\frac{B}{x+1}$$
    - Combining the fractions on the right, we get:
      $$\frac{5x-3}{x^2-2x-3}=\frac{A(x+1)}{(x-3)(x+1)}+\frac{B(x-3)}{(x-3)(x+1)}$$
    - So: $5x-3=A(x+1)+B(x-3)= Ax+A+Bx-3B=(A+B)x+(A-3B)$
    - So we get the following system of equations:
      $$\begin{aligned}A+B&=5\\A-3B&=-3\end{aligned}$$ ➔ Solving the two equations you get $A=3$, $B=2$
    - So….
      $$\int\frac{5x-3}{x^2-2x-3}dx=\int\frac{3}{x-3}+\frac{2}{x+1}dx$$
      $$=3\ln|x-3|+2\ln|x+1|+C$$

Steps to Integrating Using Partial Fractions: $\int\frac{f(x)}{g(x)}dx$

- To use this method, the degree of the top must be less than degree of the bottom.
- You also need to be able to factor $g(x)$

1. Let $x-r$ be a linear factor of $g(x)$. Suppose $(x-r)^m$ is the highest power of $x-r$ that divides $g(x)$. To this factor assign the sum of the *m* partial fractions:
   $$\frac{A_1}{x-r}+\frac{A_2}{(x-r)^2}+\frac{A_3}{(x-r)^3}+...+\frac{A_m}{(x-r)^m}$$
   Do this for each distinct linear factor of $g(x)$.

2. Let x^2+px+q be the quadratic factor of *g*. Suppose $(x^2+px+q)^n$ is the highest power of this factor that divides $g(x)$. To this factor assign the sum of the *n* partial fractions:
   $$\frac{B_1x+C_1}{x^2+px+q}+\frac{B_2x+C_2}{(x^2+px+q)^2}+\frac{B_3x+C_3}{(x^2+px+q)^3}+...+\frac{B_nx+C_n}{(x^2+px+q)^n}$$
   Do this for each distinct quadratic factor of $g(x)$ that cannot be factored into linear factors with real coefficients.
3. Set the original fraction equal to the sum of all these partial fractions. Clean up the partial fractions into one fractions using common denominators and arrange the numerator in decreasing power of *x*.
4. Set the corresponding coefficients of the numerators equal to each other and solve the system of equations.
5. Rewrite the integral using the partial fractions (which should be able to be integrated) and find the solution.

Ex. Find $\int \frac{6x+7}{(x+2)^2}\,dx$

Ex. $\int \frac{6x^2+10x+2}{x^3+3x^2+2x}\,dx$

Ex. $\int \frac{2x-4}{x^4-16}\,dx$

Exponential Model

- As we have seen, population growth is an example of Exponential Growth

$$P = P_0 e^{kt}$$

where P_0 = original population (at $t = 0$)
t = time in ears
P = population after t years
k = **Relative Growth Rate:** $k = \dfrac{dP/dt}{P}$

- k can be determined from a population table. The rate would be the ratio of the current population to the preceding year's population

Year	**Population (Millions)**	**Ratio**
1986	4936	5023/4936 ≈ 1.0176
1987	5023	5111/5023 ≈ 1.0175
1988	5111	5201/5111 ≈ 1.0176
1989	5201	5329/5201 ≈ 1.0246
1990	5329	5423/5329 ≈ 1.0175
1991	5423	

Ex. Using the table above, estimate the world population (in millions) in the year 2010.

Logistic Growth Model

- The exponential model assumes one thing: **unlimited growth**
  - This assumption is ok if the population is small
  - Eventually, the population will get to a point where it can not grow any further.
    - Therefore, the exponential model becomes no longer viable.
- A more realistic assumption is that the relative growth is positive
  - But decreases as the population increases
    - Due to:
      - Economic factors
      - Environmental factors

- The **carrying capacity**, M, is the maximum population that the environment is capable of sustaining life in the long run.
  - Assuming the relative population growth is proportional to $1-\frac{P}{M}$ with a positive constant k, then

$$\frac{dP/dt}{P}=k\left(1-\frac{P}{M}\right)$$

$$\frac{dP}{dt}=\frac{k}{M}P(M-P)$$

  - The solution to this ***logistic differential equation*** is called the **Logistic Growth Model**.
  - If P becomes higher than M, then the growth rate would be negative and the population would be decreasing.

Ex. A national park is known to be capable of supporting no more than 100 grizzly bears. Ten bears are in the park at present. We model the population with a logistic differential equation with $k = 0.1$.

a. Find a logistic growth model $P(t)$ for the population.
b. When will the population reach 50?

Homework: Pg 369-370 #1–6, 9–13, 15–21 odd, 25, 33, 35

6.3 Integration by Parts

- There are some integrals that can't be done by a simple *u*-substitution.
  - So we need another way to find these integrals.
- Suppose we take the Product Rule and write it in integral form:

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Use if you can't find a simple *u*-substitution

- Integrating both sides with respect to *x* and rearranging:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\int \frac{d}{dx}(uv)dx = \int u\frac{dv}{dx}dx + \int v\frac{du}{dx}dx$$

$$uv = \int udv + \int vdu$$

$$\boxed{\int udv = uv - \int vdu}$$

MEMORIZE!!

- This formula is called ***<u>Integration by Parts</u>***
  - This formula expresses one integral $\int udv$ in terms of another $\int vdu$
    - Choosing the correct *u* and *v* may make the second integral ($\int vdu$) easier to evaluate.

Ex. Evaluate $\int x\cos x dx$

$$\text{Let } u = x \qquad dv = \cos x dx$$

$$\frac{du}{dx} = 1 \qquad \int dv = \int \cos x dx$$

$$du = dx \qquad v = \sin x$$

$$uv - \int vdu = x\sin x - \int \sin x dx = \boxed{x\sin x + \cos x + C}$$

Check: $\frac{d}{dx}(x\sin x + \cos x + c) = x\cos x + \sin x - \sin x + 0$

$$= x\cos x$$

<u>To Set up Integration by Parts</u>:

1. You want to split this integral up into a *u* and a *dv*.
   a. *u* should be easy to find the derivative
   b. *dv* should be an easy integral
2. Find the derivative of *u* and integrate *dv*
3. Plug into the formula
   $\int udv = uv - \int vdu$
4. Simplify/evaluate

- You are trying to make the second integral easy to do. One method selecting a *u* is using the following "Order of Selection":

6. Natural Log (ln) **L**
7. Inverse Trig Function **I**
8. Polynomial Function **P**
9. Exponential Function **E**
10. Trigonometry Function **T**

L I P E T

- This is strictly a guide (works 95% of time).

Ex. Find the area bounded by the curve $y = xe^x$ and the *x*-axis from *x* = 0 to *x* = 3

$$\int_0^3 xe^x dx$$

Let $u = x$ $\qquad dv = e^x dx$

$$du = dx \qquad \int dv = \int e^x dx$$

$$u = x \qquad v = e^x$$

$$uv - \int vdu$$

$$xe^x - \int e^x dx$$

$$xe^x - e^x\Big|_0^3 = (3e^3 - e^3) - (0e^0 - e^0) = 2e^3 - (-1) = \boxed{2e^3 + 1}$$

Ex. Evaluate: $\int \ln x dx$

Note: Only 1 function – STILL USE LIPET!

Let $u = \ln x$ $dv = dx$

$\frac{du}{dx} = \frac{1}{x}$ $\int dv = \int dx$

$du = \frac{1}{x} dx$ $v = x$

$$uv - \int v du$$

$$(\ln x)(x) - \int x \cdot \tfrac{1}{x} dx$$

$$x \ln x - \int 1 dx$$

$$\boxed{x \ln x - x + C}$$

Ex. Evaluate: $\int_0^1 \tan^{-1} x dx$

$u = \tan^{-1} x$ $dv = dx$

$du = \frac{1}{1+x^2} dx$ $v = x$

$$uv - \int v du = x \tan^{-1} x - \int x \cdot \frac{1}{1+x^2} dx \longrightarrow$$

$$= \boxed{x \tan^{-1} x - \tfrac{1}{2} \ln(1+x^2) + C} \longleftarrow$$

Aside:

$\int \frac{x}{1+x^2} dx$ let $u = 1+x^2$

$du = 2x dx$

$\frac{1}{2} du = dx$

$\frac{1}{2} \int \frac{1}{u} du$

$\frac{1}{2} \ln |u|$

$\frac{1}{2} \ln |1+x^2| = \frac{1}{2} \ln(1+x^2)$

- Sometimes, you need to play around a little with the integral:

Ex. Evaluate: $\int e^x \cos x dx$

Homework: Pg. 328-329 QR #1-10, Ex. 1–8

6.3 Integration by Parts (Part 2)

Ex. Evaluate: $\int x^2 e^x dx$

Ex. Evaluate: $\int x^3 \sin x dx$

Tabular Integration

- Many Integration by parts problems are of the form

  $$\int f(x)g(x)dx$$

  - in which f can be differentiated repeatedly to become ZERO and g can be integrated repeatedly without difficulty.
- Integration by parts can result in many repetitions to get a solution
- You can organize the calculation (derivatives/integrals) and save work.

Ex. Evaluate: $\int x^2 e^x dx$

Use $f(x) = x^2$ and $g(x) = e^x$

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	(+)	e^x
$2x$	(–)	e^x
2	(+)	e^x
0		e^x

Using this table and combining the products of the functions connected by the arrows and using the alternating signs we get:

$$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c$$

Ex. Evaluate $\int x^3 \sin x dx$ using Tabular Integration

Ex. Evaluate: $\int x^4 e^{2z} dx$

Homework **Day 1: Pg. 329 #9-11, 15-20, 22, 23**
Day 2: Pg. 329 #25, 26a-d, 27, 28, 29, 31, 35